(7 pages)

Reg. No. : ....

Code No.: 6022 **Sub. Code: PMAM 41**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022

Fourth Semester

Mathematics

FUNCTIONAL ANALYSIS

(For those who joined in July 2017-2020 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The normed linear space N is a _________ space with respect to the metric d defined by $d(x, y) = \|x - y\|$

   (a) Metric (b) Complete

   (c) Hilbert (d) Inner

2. The spaces R and C the real numbers and the complex numbers are the simplest of all ______________ spaces

(a) Complex (b) Hilbert

(c) Normed linear (d) None

3. The conjugate space of N* is called as ___________conjugate.

(a) second (b) dual of N

(c) third (d) first

4. The isometric isomorphism $x \to F_x$ is called the ______of N into N**.

(a) banach (b) natural imbedding

(c) surjective (d) injuctive

5. A ―――――― space is a complex banach space whose norm arises from the inner product.

(a) Hilbert (b) Banach

(c) Inner product (d) Banach algebra

6. A ___________set in a Hilbert space H is a non empty subset of H which consists of mutually orthogonal unit vectors.

(a) Hilbert (b) Empty

(c) Orthonormal (d) Banach

7. The value of T^{**} = ——————.

(a) T (b) T^*

(c) T^1 (d) T_1

8. The operator T is self adjoint if A = ——————.

(a) A (b) A^2

(c) A^* (d) A^{**}

9. The value of det(1) = ——————.

(a) 0 (b) 1

(c) –1 (d) 2

10. $\det(T) \neq 0$ if and only if T is ________.

(a) singular (b) unitary

(c) non singular (d) self adjoint

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that if M is a closed linear subspace of a normed linear space N and x_0 is a vector not in M, then there is a functional f_0 in N^* such that $f_0(M) = 0$ and $f_0(x_0) \neq 0$.

Or

(b) Let N and N' be normed linear spaces and T a linear transformation of N into N'. Then prove that the following are equivalent.

(i) T is continuous

(ii) T is continuous at the origin

(iii) There exists a real number $K \geq 0$ with the property that $\|T(x)\| \leq K\|x\|$ for every x in N.

(iv) If S $\{x : \|x\| \leq 1\}$ is the closed unit sphere in N, then its image $T(S)$ is a bounded set in N'.

12. (a) Prove that if n is a normed linear space then the closed unit sphere S^* in N^* is a compact Hausdorff space in the weak* topology.

Or

(b) Prove that if B and B' are Banach spaces and if T is a linear transformation of B into B' then T is continuous if and only if its graph is closed.

13. (a) Prove that if M is a proper closed linear subspace of a Hilbert space H, then there exists a non zero vector z_0 in H such that $z_0 \perp M$.

Or

(b) Prove that if x and y are any two vectors in a Hilbert space H, then $|\langle x, y \rangle| \leq \|x\|\|y\|$.

14. (a) The adjoint operation $T \to T^*$ on B(H) has the following properties Prove them

(i) $(T_1 + T_2)^* = T_1^* + T_2^*$

(ii) $(\alpha T)^* = \overline{\alpha} T^*$

(iii) $(T_1 T_2)^* = T_2^* T_1^* T^{**} = T$

Or

(b) Prove that if T is an operator on H, for which $(Tx, x) = 0$ for all x then $T = 0$.

15. (a) Prove that an operator T on H is unitary if and only if it is an isometric isomorphism of H onto itself.

Or

(b) If T is normal, then prove that the Mi's are pairwise orthogonal.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Let M be a closed linear space of a normed linear space N. If the norm of a coset $x+M$ in the quotient space N/M is defined by $\|x + M\| = \inf\{\|x + m\| : m \in M\}$ then prove that N/M is a normed linear space.

Or

(b) State and prove Hahn banach theorem.

17. (a) Prove that if B and B' are Banach spaces, and if T is a continuous linear transformation of B onto B', then T is an open mapping.

Or

(b) If T is an operator on a nls N, then prove that its conjugate T^* defined by $[T^*(f)](x) = f(T(x))$ is an operator on N^* and the mapping $T \to T^*$ is an isometric isomorphism of $\mathfrak{B}(N^*)$ into $\mathfrak{B}(N^*)$ which reverses products and preserves the identity transformation.

18. (a) Prove that if M and N are closed linear subspaces of a Hubert space H such that $M \perp N$, then the linear subspace $M + N$ is also closed.

Or

(b) Prove that a closed convex subset C of a Hilbert space H contains a unique vector of smallest norm.

19. (a) Let H be a Hilbert space, and let f be an arbitrary functional in H^*, then prove that there exists a unique vector y in H such that $f(x) = (x, y)$ for every x in H.

Or

(b) Prove that if $\{e_i\}$ is an ortho normal set in a Hilbert space H, and if x is an arbitrary vector in H then $x - \sum (x, e_i) e_i \perp e_j$ for each j.

20. (a) Prove that if $B = \{e_i\}$ is a basis for H, then the mapping $T \rightarrow [T]$, which assigns to each operator T its matrix relative to B, is an isomorphism of the algebra $\mathfrak{B}(H)$ onto the total matrix algebra A_n.

Or

(b) (i) Prove that $\|N^2\| = \|N\|^2$ if N is normal operator on H.

(ii) Also prove that if T is an operator on H, then T is normal iff its real and imaginary parts commute.

(7 pages)

Reg. No. : ....

Code No. : 6023 **Sub. Code : PMAM 42**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

Fourth Semester

Mathematics – Core

COMPLEX ANALYSIS

(For those who joined in July 2017 – 2020 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. A real function of a complex variable has the derivative ____________.

   (a) Zero

   (b) ∞

   (c) Does not exist

   (d) (a) or (c)

2. If $f(x)$ is continuous then ____________ continuous.

(a) Re $f(x)$ (b) Im $f(x)$

(c) $|f(x)|$ (d) All

3. A mapping by the conjugate of an analytic function with a non-vanishing derivative is said to be

(a) Conformal (b) Direct conformal

(c) Indirectly conformal (d) Linear

4. The transformation $w = \frac{1}{z}$ is called ____________.

(a) Parallel translation

(b) Rotation

(c) Homothetic transformation

(d) Inversion

5. If $f(z)$ is analytic in an open disk Δ, then $\int_{\gamma} f(z)\,dz =$ ____________, for every closed curve γ in Δ.

(a) $= 0$ (b) $\neq 0$

(c) $2\pi i$ (d) $f(a)\,2\pi i$

6. If γ lies inside of a circle, then ____________for all points a outside of the same circle.

(a) $\eta(\gamma,a)=0$ (b) $\eta(\gamma,a) is\ a\ cons\tan t$

(c) $\eta(\gamma,a)=1$ (d) $\eta(\gamma,0)=1$

7. A function $f(z)$ which is analytic in a region Ω, except for poles is said to be ____________ in Ω.

(a) Meromorphic

(b) Isolated

(c) Isolated singularities

(d) Regular

8. The zeros of an analytic function which does not vanish identically are ____________.

(a) poles

(b) essential singularities

(c) isolated

(d) meromorphic

9. If $f(z)=\dfrac{e^z}{z^2}$ then $\text{Res}\{f(z);0\}$ is ____________.

(a) 0 (b) ∞

(c) z (d) 1

10. The residue of cotz at the simple pole $z = 0$ is ____________.

(a) 0 (b) ∞

(c) z (d) 1

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Write the statement and proof of the Lucas theorem.

Or

(b) Prove that $f(x)$ is said to have the limit A as x tends to a, $\lim_{lx \to a} f(x) = A$ iff the following is true for every $\in > 0$ there exists a number $\delta > 0$ with the property that $|f(x) - A| < \in$ for all values of x such that $|x - a| < \delta$ and $x \neq a$.

12. (a) Show that the line integral $\int_{\gamma} p\,dx + q\,dy$ defined in Ω depends only on the end point of γ iff there exists a function $U(x, y)$ in Ω with the partial derivative $\frac{\partial U}{\partial x} = p, \frac{\partial U}{\partial y} = q$.

Or

(b) Find the linear transformation which carries $0, i, -i$ in to $1, -1, 0$.

13. (a) Prove that if the piecewise differential closed curve γ does not pass through the point a, then the value of the integral $\int_{\gamma} \frac{dz}{z-a}$ is a multiple of $2\pi i$.

Or

(b) Prove that $\int_{C} \frac{dz}{z^2+1} = 0$ where C is the positively oriented circle $|z| = 2$.

14. (a) Show that the functions e^z, $\cos z$ and $\sin z$ have essential singularities at ∞.

Or

(b) State and prove Liouville's theorem.

15. (a) Compute the value of $\int_0^{\infty} \frac{x \sin x}{x^2 + a^2} dx$, a real

Or

(b) State and prove Rouche's theorem.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove Abel's limit theorem.

Or

(b) State and prove Abel's theorem by the convergence.

17. (a) Prove that the cross ratio (z_1, z_2, z_3, z_4) is real iff the four points lie on a circle or on a straight line.

Or

(b) Investigate the geometric significance of symmetric.

18. (a) State and prove Cauchy's theorem for rectangle.

Or

(b) Suppose that $f(z)$ is analytic in an open disc Δ, and let γ be a closed curve in Δ for any point a not on γ then prove that $n(\gamma, a)$ $f(a) = \frac{1}{2\pi i} \int_{\gamma} \frac{f(z)\,dz}{z-a}$ where $n(\gamma, a)$ is the index of a with respect to γ.

19. (a) Write an elaborate proof for the following: suppose that $\phi(\zeta)$ is continuous on the arc γ. Then show that the function $F_n(z) = \int_\gamma \frac{\phi(\zeta)\, d\zeta}{(\zeta - z)^n}$ is analytic in each of the regions determined by γ. And its derivative is $F'_n(z) = nF_{n+1}(z)$.

Or

(b) Sate and prove Taylor's theorem.

20. (a) State and prove Residue theorem.

Or

(b) Determine the value of

$$\int_{-\infty}^{\infty} \frac{\cos x}{(x^2 + a^2)(x^2 + b^2)} dx \quad (a > b > 0)$$

———————

(6 pages) **Reg. No. :** ....

Code No. : 6024 **Sub. Code : PMAM 43**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Fourth Semester

Mathematics — Core

ADVANCED ALGEBRA — II

(For those who joined in July 2017–2020 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. A field K is said to be an extension of F if ————

   (a) $K \supseteq F$ (b) $K \subseteq F$

   (c) $K \in F$ (d) $F \notin K$

2. A complex number is said to be an ———— number if it is algebraic over the field of rational numbers.

   (a) extension (b) algebraic

   (c) monic (d) filed

3. If $p(x) \in F[x]$ then an element a in some extension field of F is called a ———— of $p(x)$ if $p(a) = 0$.

(a) degree (b) dual

(c) root (d) first

4. The element $a \in K$ is a root of $p(x) \in F[x]$ of ———— m if $(x-a)^m / p(x)$ and $(x-a)^{m+1} \nmid p(x)$

(a) degree (b) root

(c) order (d) multiplicity

5. If G is a group of automorphism of K, then the fixed field of G is the set of all elements $a \in K$ such that ———— for all $a \in G$.

(a) $a = 0$ (b) $K = c$

(c) $\sigma(x) = 0$ (d) $\sigma(a) = a$

6. K is a ———— extension of F if k is a finite extension of F such that F is the fixed field of $G(K, F)$.

(a) order (b) inverse

(c) normal (d) simple

7. The multiplicative group of non zero elements of a finite field is ———

(a) multiplicity (b) cyclic

(c) finite (d) normal

8. For every prime number p and every positive integer m, there is a unique field having ——— elements.

(a) p^m (b) m

(c) p^{-m} (d) p

9. If $x = \alpha_0 + \alpha_1 i + \alpha_2 j + \alpha_3 k$ in K then $x^* = \alpha_0 - \alpha_1 i + \alpha_2 j - \alpha_3 k$ is ——— of x.

(a) adjoint (b) normal

(c) extension (d) regular

10. If $x \in Q$ then the ——— of x is $N(x) = xx^*$.

(a) absolute value (b) norm

(c) zero (d) 1

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that if L is an algebraic extension of K and if K is an algebraic extension of F, then L is an algebraic extension of F.

Or

(b) Prove that if $a \in K$ is algebraic of degree n over F, then $[F(a) : F] = n$.

12. (a) State and prove remainder theorem.

Or

(b) Prove that a polynomial of degree n over a field can have at most n roots in any extension field.

13. (a) Prove that if K is a normal extension of F if and only if K is the splitting field of some polynomial over F.

Or

(b) Prove that the fixed field of G is a subfield of K.

14. (a) Prove that for ever prime number p and every positive integer m their exists a field having p^m elements.

Or

(b) Let D be a division ring such that for every $a \in D$ there exists a positive integer $n(a) > 1$, depending on a, such that $a^{n(a)} = a$ then D is a commutative field.

15. (a) Prove that for all $x, y \in Q$, $N(xy) = N(x)N(y)$.

Or

(b) Prove that if $a \in H$ then $a^{-1} \in H$ if and only if $N(a) = 1$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Prove that if L is a finite extension of K and if K is a finite extension of F, then L is a finite extension of F. More over $[L : F] = [L : K];[K : F]$.

Or

(b) Prove that the element $a \in K$ is algebraic over F if and only if $F(a)$ is a finite extension of F.

17. (a) Prove that if F is of characteristic zero and if a,b are algebraic over F, then there exists and element $c \in F(a,b)$ such that $F(a,b) = F(c)$.

Or

(b) Prove that if $p(x)$ is irreducible in $F[x]$ and if v is a root of $p(x)$, then $F(v)$ is isomorphic to $F'(w)$ where w is a root of $p'(t)$. This isomorphism α can so be chosen that 1. $v\sigma = w$ and $\alpha\sigma = \alpha'$ for every $\alpha \in F$.

18. (a) Let K be a normal extension of F and let H be a subgroup of $G(K,H)$; let $K_H = \{x \in K / \sigma(x) = x \; for\, all\, \sigma \in H\}$ be the fixed field of H. Then prove that (i) $[K : K_H] = o(H)$ (ii) $H = G(K, K_H)$.

Or

(b) Prove that if K is a finite extension of F, then $G(K,F)$ is a finite group and its order, $o(G(K,F))$ satisfies $o(G(K,F)) \leq [K : F]$.

19. (a) Prove that if G be a finite abelian group enjoying the property that the relation $x^n = e$ is satisfied by at most n elements of G, for every integer n then G is a cyclic group.

Or

(b) Prove that a finite division ring is necessarily a commutative field.

20. (a) State and prove Frobenius theorem.

Or

(b) Prove that every positive integer can be expressed as the sum of squares of four integers.

(6 pages)

Reg. No. : ....

Code No. : 6025 **Sub. Code : PMAM 44**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

Fourth Semester

Mathematics — Core

TOPOLOGY — II

(For those who joined in July 2017–2020)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. A space for which every open covering contains a countable sub covering is called a

   (a) Compact space (b) Lindelof space

   (c) Separable space (d) Normal space

2. Consider the two statements

A : the space R_l is normal

B : the space R_l^2 is normal. then

(a) Both A and B are true

(b) Neither A nor B is true

(c) A is true but B is not true

(d) A is not true but B is true

3. Which one of the following <u>not</u> *true*

(a) Every metrizable space is normal

(b) Every compact Hausadroff space is normal

(c) Every regular space is normal

(d) Every well-ordered set is normal in the order topology

4. Urysohn lemma asserts the existence of certain real - valued continuous functions on a ———— space X.

(a) Hausdorff (b) Regular

(c) Completely regular (d) Normal

5. Every regular space X with a ———— is metrizable.

(a) countable basis (b) dense subset

(c) basis (d) continuous function

6. A space is completely regular if and only if it is homemorphic to a subspace of

(a) R^w

(b) $[0,1]^J$ for some J

(c) $[a,b]$ for some a and b with $a<b$

(d) a Lindelof space

7. "An arbitrary product of compact spaces is compact is the product topology" - this result is known as

(a) Urysohn lemma

(b) Tychonoff theorem

(c) Tietze extension theorem

(d) Urysohn metrization theorem

8. Consider the statements

A : $\{(n, n+2 / n \in Z\}$ is locally finite $\mathbb{R}$

B : $\{(0, 1/n)/n \in Z_+\}$ is locally finite in (0, 1) then

(a) Both A and B are true

(b) Neither A nor B is true

(c) A is true but B is not true

(d) A is not time but B is true

9. The interior of $[0, 1] \times 0$ as a subset of the plane $\mathbb{R}^2$ is

(a) $(0, 1) \times 0$ (b) ϕ

(c) $(0, 1) \times (0, 1)$ (d) $[0, 1] \times 0$

10. Which one of the following is not true

(a) The space Q of rationals is a Baire space

(b) The space Z_+ is a Baire space

(c) Every closed subspace of R is a Baire space

(d) The irrationals in $\mathbb{R}$ from a Baire space

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).
Each answer should not exceed 250 words.

11. (a) Give an example of a topological space which satisfies the first countability axiom but does not satisfy the second.

Or

(b) Prove that a product of Hausdorff space is Hausdorff.

12. (a) Show that every metrizable space is normal.

Or

(b) Define a completely regular space and show that a subspace of a completely regular space is completely regular.

13. (a) Give an example showing that a Hausdorff space with a countable basis need not be metrizable.

Or

(b) Show that the Tietze extension theorem implies the Urysohn lemma.

14. (a) Define a locally finite collection of subsets in a topological space with an example.

Or

(b) Let A be a locally finite collection of subsets of X. Prove that $\overline{\underset{A\in A}{U} A} = \underset{A\in A}{U} \overline{A}$.

15. (a) Show that the irrational are a Baire space.

Or

(b) Prove that any open subspace of a Baire space is itself a Baire space.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b)
Each answer should not exceed 600 words.

16. (a) Show that the product of two Lindelot spaces need not be Lindelot.

Or

(b) Show that the space $\mathbb{R}_K$ is Hausdorff but not regular.

17. (a) Show that every well ordered set X is normal in the order topology.

Or

(b) State and prove the Urysohn lemma.

18. (a) State and prove Urysohn metrization theorem.

Or

(b) State and prove Tietze extension theorem.

19. (a) Prove that an Arbitraz product of compact spaces is compact in the product topology.

Or

(b) Let X be a metrizable space. If A is an open covering of X, prove that there is an open covering $\in$ of X refining of that is countably locally finite.

20. (a) State and prove baire category theorem.

Or

(b) Let X be a space; let (Y, d) be a metric space. Let $f_n : X \to Y$ be a sequence of continuous functions such that $f_n(x) \to f(x)$ for all $x \in X$, where $f : X \to Y$. If X is a baire space, prove that the set of prints at which f is continuous is dense in X.

(6 pages) **Reg. No. :** ....

Code No. : 6364 **Sub. Code : ZMAM 11**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

First Semester

Mathematics — Core

ALGEBRA — I

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. A homomorphism ϕ from G into $\overline{G}$ is said to be an isomorphism if ϕ is ——————

   (a) one to one (b) unto

   (c) not one to one (d) bijective

2. Every subgroup of an abelian group is ——————

   (a) right coset (b) last coset

   (c) normal (d) not normal

3. In a group $b^5 = e$ and $aba^{-1} = a^2$ for some $a, b \in G$. The order of a is ————

(a) 5 (b) 10

(c) 0 (d) divisor of 10

4. Let G be a group and ϕ an automorphism of G. If $a \in G$ is of order $o(a) > 0$, then $o(\phi(a)) =$ ————

(a) 0 (b) 1

(c) $o(a)$ (d) ∞

5. If $\alpha = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$ then $\alpha\beta =$ ————

(a) $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix}$

6. If $o(G) = p^2$ where p is a prime number then G is ————

(a) normal (b) left coset

(c) right coset (d) abelian

7. The value of $9c_2$ is ————

(a) 18 (b) 8

(c) 32 (d) 36

8. The number of p-sylow subgroups in G, for a given prime is of the form ————

(a) $1+kp$ (b) $1-kp$

(c) kp (d) $\frac{1+k}{p}$

9. If $\phi \neq 1 \in G$ where G is an abelian group then $\sum_{g \in G} \phi(g) =$ ————

(a) 1 (b) 2

(c) ∞ (d) 0

10. The number of non-isomorphic abelian groups of order p^n, p an prime, equals the number of partitions of ————.

(a) $\frac{n}{2}$ (b) $n!$

(c) n (d) $n-1$

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If G is a finite group and N is a normal subgroup of G, then prove that $o(G/N) = o(G)/o(N)$.

Or

(b) If ϕ is a homomorphism of G into $\overline{G}$, then prove that :

(i) $\phi(e) = \overline{e}$, the unit element of $\overline{G}$.

(ii) $\phi(x^{-1}) = \phi(x)^{-1}$ for all $x \in G$

12. (a) Show that $\mathcal{I}(G) \approx G/Z$, where $\mathcal{I}(G)$ is the group of inner automorphisms of G, and Z is the center of G.

Or

(b) If H is a subgroup of G show that for every $g \in G$, gHg^{-1} is a subgroup of G.

13. (a) Prove that $N(a)$ is a subgroup of G.

Or

(b) If $o(G) = p^n$ where p is a prime number, then prove that $Z(G) \neq (e)$.

14. (a) Prove that $n(k) = 1 + p + \ldots + p^{k-1}$.

Or

(b) If $p^m \mid o(G), p^{m+1} \nmid o(G)$, then prove that G has a subgroup of order p^m.

15. (a) Let G be a group and suppose that G is the integral direct production of $N_1,....,N_n$. Let $T = N_1 \times N_2 \times ... \times N_n$. Then prove that G and T are isomorphic.

Or

(b) If G and G' are isomorphic abelian groups, then prove that for every integer s, $G(s)$, and $G'(s)$ are isomorphic.

PART C — $(5 \times 8 = 40$ marks)

Answer ALL questions choosing either (a) or (b).

16. (a) State and prove Sylow's theorem for Abelian groups.

Or

(b) Let ϕ be a homomorphism of G onto $\overline{G}$ with kernel K, and let $\overline{N}$ be a normal subgroup of $\overline{G}, N = \{x \in G \mid \phi(x) \in \overline{N}\}$. Then prove that $G/N \approx \overline{G}/\overline{N}$. Equivalently. $G/N \approx (G/K)/(N/K)$.

17. (a) If G *is* a group, then prove that $\mathcal{A}(G)$, the set of automorphisms of G, is also a group.

Or

(b) Let G be a finite group, T an automorphism of G with the property that $xT = x$ iff $x = e$. Suppose further that $T^2 = 1$ prove that G must be abelian.

18. (a) State and prove Cauchy theorem.

Or

(b) Prove : $o(G)=\sum \frac{o(G)}{o(N(a))}$ where this sum runs over one element a in each conjugate class.

19. (a) State and prove Sylow theorem.

Or

(b) Prove that S_{p^k} has a p-sylow subgroup.

20. (a) Let G be an abelian group of order p^n, p a prime. Suppose that $G=A_1 \times A_2 \times \ldots \times A_k$, where each $A_i=(a_i)$ is cyclic of order p^{n_i}, and $n_1 \geq n_2 \geq \ldots \geq n_k > 0$. If m is an integer such that $n_t > m \geq n_{t+1}$ then prove that $G(p^m)= B_1 \times \ldots \times B_t \times A_{t+1} \times \ldots . \times A_k$ where B_i is cyclic of order p^m, generated by $a_i^{p^{n_i-m}}$, for $i \leq t$. The order of $G(p^m)$ is p^u, where $u=mt\sum_{i=t+1}^{k} n_i$.

Or

(b) Show that the two abelian groups of order p^n are isomorphic iff they have the same invariants.

(8 pages) **Reg. No. :** ....

Code No. : 6365 **Sub. Code : ZMAM 12**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

First Semester

Mathematics — Core

ANALYSIS — I

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. Any discrete metric space is ——————

   (a) first category (b) second category

   (c) third category (d) none of these

2. Any discrete metric space having more than one point is ——————

   (a) connected (b) finite

   (c) null set (d) disconnected

3. If the sequence $\{a_n\}$ is bounded and sequence $\{b_n\}$ converges to zero then the sequence $\{a_n\, b_n\}$ ____________

(a) diverges to $+\infty$ (b) diverges to $-\infty$

(c) converges to zero (d) none of these

4. Find lim sup a_n for the sequence $\{a_n\} = \{n!\}$ ____________

(a) 1 (b) 0

(c) ∞ (d) none of these

5. Applying Cauchy's root test the series $\sum_{n=1}^{\infty}\left(\frac{n}{2n+1}\right)^n$ is ____________

(a) convergent

(b) divergent

(c) neither convergent nor divergent

(d) both convergent and divergent

6. If the n^{th} term of a series is $a_n = \frac{1.2.3\cdots n}{3.5.7\cdots 2n-1}$ then $\lim_{n\to\infty}\frac{a_n}{a_{n+1}} =$ ____________

(a) 1 (b) 2

(c) $\frac{1}{2}$ (d) 0

7. Which of the following is equivalent to compactness in a metric space M?

(a) M is totally bounded

(b) M is complete

(c) Every bounded subset of M has a limit point

(d) Every infinite subset of M has a limit point

8. Which of the following subset of R is both compact and connected? ————

(a) R (b) (0, 1)

(c) [0,100] (d) Q

9. Let f be defined on $[a, b]$; if f has a local maximum at a point $x \in (a,b)$, and if $f'(x)$ exists, then $f'(x)$ ————

(a) 1 (b) 2

(c) 0 (d) ∞

10. Suppose f is differentiable in (a, b) if $f'(x) = 0$ or all $x \in (a,b)$, then f is ————

(a) monotonically increasing

(b) monotonically decreasing

(c) constant

(d) none of these

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that the closed subsets of compact sets are compact.

Or

(b) Let K be a positive integer. If $\{I_n\}$ is a sequence of k-cells such that $I_n \supset I_{n+1} (n = 1, 2, 3, \cdots)$, then prove that $\bigcap_1^\infty I_n$ is not empty.

12. (a) Show that if $p > 1$, $\sum_{n=2}^{\infty} \frac{1}{n (\log n)^p}$ converges; if $p \leq 1$, the series diverges.

Or

(b) Suppose $\{S_n\}$ is monotonic. Then prove that $\{S_n\}$ converges iff it is bounded.

13. (a) If $\sum a_n = A$, and $\sum b_n = B$, then prove that $\sum (a_n + b_n) = A + B$, and $\sum ca_n = CA$ for any fixed *c*.

Or

(b) Prove :

(i) the partial sums A_n of $\sum a_n$ from a bounded sequence;

(ii) $b_0 \geq b_1 \geq b_2 \geq \cdots$;

(iii) $\lim_{n \to \infty} b_n = 0$.

14. (a) Let f be monotonic on (a, b). Then prove that the set of points of (a, b) at which f is discontinuous is at most countable.

Or

(b) Suppose f is a continuous mapping of a compact metric space X into a metric space Y then. Prove that $f(X)$ is compact.

15. (a) If f and g are continuous real function on $[a, b]$ which are differentiable in (a, b), then prove that there is a point $x \in (a, b)$ at which $[f(b) - f(a)] g'(x) = [g(b) - g(a)] f'(x)$. Note that differentiability is note required at the end points.

Or

(b) Suppose f is a real differentiable function on $[a, b]$ and suppose $f'(a) < \lambda < f'(b)$. Then prove that there is a point $x \in (a, b)$ such that $f'(x) = \lambda$. A similar result holds of course if $f'(a) > f'(b)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Prove that a subset E of the real line R^1 is connected iff it has the following property: If $x \in E, Y \in E$, and $x < z < y$, then $z \in E$.

Or

(b) Suppose $K \subset Y \subset X$. Then prove K is compact relative to X iff K is compact relative to Y.

17. (a) Prove that the following :

(i) If $\{p_n\}$ is a sequence in a compact metrix space X, then some subsequence of $\{p_n\}$ converges to a point of X.

(ii) Every bounded sequence in R^k contains a convergent subsequence.

Or

(b) Prove that e is irrational

18. (a) Suppose

(i) $\sum_{n=0}^{\infty} a_n$ converges absolutely

(ii) $\sum_{n=0}^{\infty} a_n = A$,

(iii) $\sum_{n=0}^{\infty} b_n = B$,

(iv) $C_n \sum_{k=0}^{n} a_k \, b_{n-k} \, (n = 0, 1, 2, \cdots)$.

Then prove that $\sum_{n=0}^{\infty} C_n = AB$. That is, the product of two convergent series converges, and to the right value, if at least one of the two series converges absolutely.

Or

(b) State and prove Ratio Test.

19. (a) Let f be a continuous mapping of a compact metric space X into a metric space Y. Then prove that f uniformly continuous on X.

Or

(b) Let X, Y, E, f, and p is a limit point of E. Then prove that $\lim_{x \to p} f(x) = q$ iff $\lim_{n \to \infty} f(p_n) = q$ for every sequence $\{p_n\}$ in E such that $p_n \neq p, \lim_{n \to \infty} p_n = p$.

20. (a) Suppose f is continuous on $[a, b]$, $f'(x)$ exists at some point $x \in [a, b]$, g is defined on an interval I which contains the range of f, and g is differentiable at the point $f(x)$. If $h(t) = g(f(t)) (a \le t \le b)$, then prove that h is differentiable at x, and $h'(x) = g'(f(x)) f'(x)$.

Or

(b) State and prove that L' Hospital's rule.

(7 pages) **Reg. No. :** ....

Code No.: 6366 **Sub. Code: ZMAM 13**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

First Semester

Mathematics – Core

ANALYTIC NUMBER THEORY

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If $n \mid n$, then it is called __________ property of divisibility.

   (a) reflexive

   (b) symmetric

   (c) transitivity

   (d) linearity

2. The series $\sum_{n=1}^{\infty} 1/p_n$ is ____________

(a) converges (b) diverges

(c) countable (d) uncountable

3. The value $\varphi(8) =$ ____________

(a) 1 (b) 2

(c) 4 (d) 0

4. The notation for Mobius function is ____________.

(a) $\varphi(n)$ (b) $\pi(n)$

(c) $f(n)$ (d) $\mu(n)$

5. The identity function $I(n) = [1/n]$ is ____________.

(a) not multiplicative

(b) multiplicative

(c) completely multiplicative

(d) not complete

6. If any two functions f and g are multiplicative, then ____________ multiplicative

(a) fg (b) f/g

(c) none of the above (d) both

7. The value of $\lim_{x\to\infty} \frac{\pi(x)\log x}{x} =$ ______________.

(a) 1 (b) 2

(c) 4 (d) 0

8. The average order of $\Lambda(n)$ is _________

(a) 2 (b) 1

(c) 4 (d) 0

9. The upper bound of $\pi(n) =$ ______________.

(a) $\frac{1}{6}\frac{n}{\log n}$ (b) $\frac{n}{\log n}$

(c) $6\frac{n}{\log n}$ (d) $2\frac{n}{\log n}$

10. The value of $\lim_{x\to\infty} \frac{\log x}{\log \pi(x)} =$ ______________.

(a) 0 (b) 2

(c) 4 (d) 1

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that every integer $n > 1$ is either a prime number or a product of prime numbers.

Or

(b) State and prove division algorithm.

12. (a) Prove that $\varphi(mn) = \varphi(m)\varphi(n)(d, \varphi(d))$ where $d = (m, n)$. Also prove that $\varphi(a) \mid \varphi(b)$ if $a \mid b$.

Or

(b) State and prove Mobius inversion formula.

13. (a) Given f with $(1) = 1$. Then prove that f is multiplicative if and only if $f\left(p_1^{a_1}, p_2^{a_2}, \ldots, p_r^{a_r}\right) = f\left(p_1^{a_1}\right)f\left(p_2^{a_2}\right)\ldots f\left(p_r^{a_r}\right)$ for all primes p_i and all integers $a_i \geq 1$.

Or

(b) State and prove Generalized inversion formula.

14. (a) If $x \geq 1$, then prove that $\sum_{n \leq x} \frac{1}{n} = \log x + C + 0\left(\frac{1}{x}\right)$. Also prove that $\sum_{n \leq x} n^{\alpha} = \frac{x^{\alpha+1}}{\alpha + 1} + 0\left(x^{\alpha}\right)$ if $\alpha \geq 0$.

Or

(b) For all $x > 1$, show that $\sum_{n \leq x} \varphi(n) = \frac{3}{\pi^2} x^2 + 0(x \log x)$.

15. (a) For all $x \geq 1$, prove that $\left|\sum_{n \geq x} \frac{\mu(n)}{n}\right| \leq 1$ with equality holding only if $x < 2$.

Or

(b) For $x \geq 2$, show that $\sum_{p \leq x} \left[\frac{x}{p}\right] \log p = x \log x + 0(x)$ where the sum is extended over all primes $\leq x$.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b)

16. (a) (i) State and prove Euclidean algorithm.

(ii) If $(a,b) = 1$, then prove that $(a^n, b^k) = 1$ for all $n \geq 1$, $k \geq 1$.

Or

(b) (i) Prove that $n^4 + 4$ is composite if $n > 1$.

(ii) Prove that every integer $n > 1$ can be represented as a product of prime factors in only one way, apart from the order of the factors.

17. (a) State and prove the product formula for $\varphi(n)$.

Or

(b) Define Mobius function and find the relationship between Mobius function and Euler totient function.

18. (a) Define Liouville's function and for every $n \geq 1$ and prove that $\sum_{d|n} \lambda(d) = \begin{cases} 1, & \text{if } n \text{ is a square} \\ 0, & \text{otherwise} \end{cases}$.

Or

(b) State and prove Generalized Mobius inversion formula.

19. (a) Prove that the set of lattice points visible from the origin has density $6/\pi^2$.

Or

(b) For all $x \geq 1$ and $\alpha > 0$, $\alpha \neq 1$, prove that,

(i) $\sum_{n \leq x} \sigma_1(n) = \frac{1}{2}\zeta(2)x^2 + 0(x \log x)$

(ii) $\sum_{n \leq x} \sigma_\alpha(n) = \frac{\zeta(\alpha+1)}{\alpha+1} x^{\alpha+1} + 0(x^\beta)$ where $\beta = \max\{1, \alpha\}$.

20. (a) For $n \geq 1$, prove that the n^{th} prime p_n satisfies the inequality $\frac{1}{6} n \log n < p_n < 12\left(n \log n + n \log \frac{12}{e}\right)$.

Or

(b) Prove that the following relations are logically equivalent

(i) $\lim_{x\to\infty} \dfrac{\pi(x)\log x}{x} = 1$

(ii) $\lim_{x\to\infty} \dfrac{\vartheta(x)}{x} = 1$

(iii) $\lim_{x\to\infty} \dfrac{\psi(x)}{x} = 1$

Reg. No. : ....

Code No. : 6367 **Sub. Code : ZMAM 14**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

First Semester

Mathematics — Core

OPERATIONS RESEARCH

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The unit "transportation" cost from period i to period j is computed as $C_{ij} =$

   (a) Production cost is i, $i = j$

   (b) Production cost is i + holding cost from i to j, $i < j$

   (c) Production cost is i + penality cost from to i to j, $i < j$

   (d) All the above

2. Which method yields the best starting solutions of the transportation problem

(a) North west-corner method

(b) Least-cost method

(c) Vogel approximation method

(d) None

3. A circuit is a loop in which all the branches are oriented in the ———.

(a) Opposite direct (b) Same direction

(c) Both direction (d) None

4. Total float of an activity is $TF_{ij} =$ ———.

(a) $LC_j - ES_i - D_{ij}$ (b) $LC_j + ES_i - D_{ij}$

(c) $LC_j - ES_i + D_{ij}$ (d) none

5. Which one of the following is IP

(a) Zero-one (b) Mixed zero -one

(c) Pure integer (d) Mixed

6. Additive algorithms required presenting the 0–1 problem in a convenient form that satisfies ——— requirement

(a) 1 (b) 2

(c) 3 (d) none

7. During the classic EOQ model, the reorder point occurs when the ———— to LD units.

(a) inventory level drops

(b) inventory level increases

(c) all the above (a) and (b)

(d) none

8. In constant rate demand with instantaneous replenishment and no shortage model $Y^* =$ ————

(a) $\frac{DK}{Y} + \frac{YK}{2}$ (b) $\sqrt{\frac{2DK}{h}}$

(c) $\frac{Yh}{2}$ (d) $\sqrt{2DKh}$

9. The expected waiting time in the model $(M/M/I)$: $(G_D/\infty/\infty)$ is

(a) $\frac{\rho}{1-\rho}$ (b) $\frac{1}{\mu(1-\rho)}$

(c) $\frac{\rho}{\mu(1-\rho)}$ (d) $\frac{\rho^2}{1-\rho}$

10. In $(M/M/\infty):(G_D/\infty/\infty)$ model $P_0 =$ ————.

(a) $1-\rho$ (b) $e^{-\rho}$

(c) $\frac{1}{\mu}$ (d) $\lambda\mu$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that in a symmetric TSP, any three distinct cost elements $C(i\ j), C(j, k), C(k,i)$ can be set to infinity without eliminating a minimum length tours.

Or

(b) Solve the 4-city TSP whose distance matrix is given in the following table :

City	1	2	3	4
1	–	12	10	14
2	12	–	13	8
3	10	13	–	12
4	14	8	12	–

12. (a) Explain Maximal Flow algorithm.

Or

(b) Explain Dijkstra's algorithm.

13. (a) Solve the following zero-one problem using implicit enumeration algorithm.

Maximize $4x_1 + 3x_2 - 2x_3$

Subject to

$x_1 + x_2 + x_3 \leq 8$

$2x_1 - x_2 - x_3 \leq 4$

$x_2, x_3 = 0, 1$

Or

(b) Solve the MILP :

Maximize $2x_1 + 3x_2$

Subject to

$3x_1 + 4x_2 \leq 10$

$x_1 + 3x_2 \leq 7$

$x_1 \geq 0$

$x_2 \geq 0$

and are integers.

14. (a) An item is consumed at the rate of 30 items per day. The holding cost per unit per day is \$. 05 and the set up cost in \$ 100. Suppose that no shortage is allowed and that the purchasing cost per unit is \$10 for any quantity not exceeding 500 units and \$ 8 otherwise. Determine the optimal inventory policy given a 21-day lead time.

Or

(b) A music store sells a best-selling compact disc. The daily demand for the disc in approximately normally distributed with mean 200 disc and a standard deviation of 20 disc. The cost of keeping the disc in the store is \$. 04 per disc per day. It cost the store \$ 100 to place a new order. The supplier normally specifies a 7-day lead time for delivery. Assuming that the store wants so limit the probability of running out of disc during the lead time to no more than .02, determine the stores optimal inventory policy.

15. (a) Consider the production - consumption inventory model with back orders. The data are D = 10000 / year, P = 16000 / year, C_0 = 350 / set up, C_c = 3.6 / unit / year and D = 100 unit / year. Find the batch quantity Q and the total cost.

Or

(b) Explain the model $(M/M/1):(GD/\infty/\infty)$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Solve the transportation model.

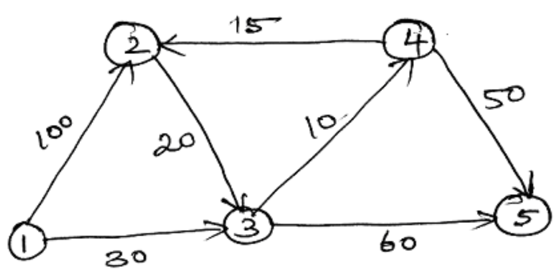
$	0	2	1	6
	2	1	5	7
	2	4	3	7
	5	5	10	

Or

(b) Solve the assignment model

1	4	6	3
9	7	10	9
4	5	11	7
8	7	8	5

17. (a) *D* Find the shortest path from 1 to 5 using Dijkstra's algorithm.



Or

(b) Discuss the computations of critical path method.

18. (a) Solve the following 0-1 problem

Maximize $w = 3y_1 + 2y_2 - 5y_3 - 2y_4 + 3y_5$

Subject to

$y_1 + y_2 + y_3 + 2y_4 + y_5 \leq 4$
$7y_1 + 3y_3 - 4y_4 + 3y_5 \leq 8$
$11y_1 - 6y_2 + 3y_4 - 3y_5 \geq 3$
$y_1, y_2, y_3, y_4, y_5 = (0, 1)$

Or

(b) Solve the following by using fractional cut

Maximize $z = 3x_1 + x_2 + 3x_3$

Subject to :

$x_1 + 2x_2 + x_3 \leq 4$
$4x_2 - 3x_3 \leq 2$
$x_1 - 3x_2 + 2x_3 \leq 2$
$x_1, x_2, x_3 \geq 0$

and are integers.

19. (a) The following data describe four inventory items. The company wishes to determine the economic order quantity for each of the four items such that the total number of orders per year (365 days) is atmost 150.

Item	K_i	D_i	h_i
	\$	units/d	
1	100	10	.1
2	50	20	2
3	90	5	.2
4	20	10	.1

Or

(b) Explain the model Multi-item with storage Limitations.

20. (a) Explain $(M/M/C):(GD/N/\infty),\ C \leq N$.

Or

(b) Patients arrive at a clinic according to a Poisson distribution at the rate of 20 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patients is exponential with mean of 8 minutes

(i) What is the probability that an arriving patient will not wait?

(ii) What is the probability that a patient to find a vacant seat in the room?

(iii) What is the expected waiting time until a patient leaves the clinic?

(7 pages) **Reg. No. :** ....

Code No. : 6368 **Sub. Code : ZMAM 15**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

First Semester

Mathematics – Core

ORDINARY DIFFERENTIAL EQUATION

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Which of the following second order non-homogenous linear differential equation?

(a) $\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)y^2=R(x)$

(b) $\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)y^2=0$

(c) $\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)y=R(x)$

(d) $\frac{d^2y}{dx^2}+P(x)\frac{dy}{dx}+Q(x)y=0$

2. Any linear combination of two solution of the homogeneous equation is ____________

(a) Not a solution (b) Solution

(c) Linear (d) Non-linear

3. If $f(x)$ and $g(x)$ are analytic at x_0, then ____________ is analytic.

(a) $f(x)+g(x)$ (b) $f(x)g(x)$

(c) $\frac{f(x)}{g(x)}$ if $g(x_0)\neq 0$ (d) All the above

4. The series $\sum_{n=0}^{\infty}\frac{x^n}{n!}$ ____________

(a) converges for all x

(b) converges only at $|x|<1$

(c) diverges at $|x|>1$

(d) diverges for all $x\neq 0$

5. Which of the following series is Frobenius series?

(a) $y=a_0+a_1x^m+a_2x^{(m+1)}+\ldots$

(b) $y=a_0x^m+a_1x^{m+1}+a_2x^{m+2}+\ldots$

(c) $y=x^m(a_0+a_2+a_1+\ldots)$

(d) $y=x^m(a_0x+a_1x^2+a_2x^3+\ldots)$

6. If the function $(x-x_0)P(x)$ and $(x-x_0)^2Q(x)$ are not analytic then singular point x_0 is said to be ____________

(a) Irregular (b) Regular

(c) Singular (d) Non-Singular

7. $J_{\frac{-1}{2}}(x)=$

(a) $\sqrt{\frac{2}{\pi x}}\cos x$ (b) $\sqrt{\frac{1}{\pi x}}\cos x$

(c) $\sqrt{\frac{2}{\pi x}}\sin x$ (d) $\sqrt{\frac{1}{\pi x}}\sin x$

8. $\Gamma(p)=$

(a) $\Gamma(p+1)$ (b) $\Gamma(p-1)$

(c) $\frac{\Gamma(p+1)}{p}$ (d) $p\Gamma(p+1)$

9. The value of $\begin{vmatrix} e^{3t} & e^{2t} \\ e^{3t} & 4e^{2t} \end{vmatrix}$

(a) e^{4t} (b) $3e^{5t}$

(c) $2e^{5t}$ (d) $3e^{4t}$

10. The auxiliary question of $\begin{cases} \dfrac{dx}{dt} = 3x - 4y \\ \dfrac{dy}{dt} = x - y \end{cases}$ is

(a) $(m-1)^2 = 0$ (b) $(m+1)^2 = 0$

(c) $m^2 + 2m - 1 = 0$ (d) $m^2 - 2m - 1 = 0$

PART B — $(5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that if $y_1(x)$ and $y_2(x)$ are any two solution of $y'' + P(x)y' + Q(x)y = 0$, then $c_1 y_1(x) + c_2 y_2(x)$ is also a solution for any constants c_1 and c_2.

Or

(b) Show that $y = c_1 x + c_2 x^2$ is the general solution of $x^2 y'' - 2xy' + 2y = 0$ on any interval not containing 0, and find the particular solution for which $y(1) = 3$ and $y'(1) = 5$.

12. (a) Find the general solution of $(1 + x^2)y'' + 2xy' - 2y = 0$ in terms of power series in x.

Or

(b) Find the power series for $\dfrac{1}{(1-x)^2}$ by squaring.

13. (a) Find two independent Frobenius series solutions of $x^2y'' - x^2y' + (x^2 - 2)y = 0$.

Or

(b) Determine the nature of the point $x = 0$ for $y'' + (\sin x)y = 0$.

14. (a) Show that $\frac{d}{dx}[xJ_1(x)] = xJ_0(x)$.

Or

(b) Prove that $\frac{d}{dx}[x^pJ_p(x)] = x^pJ_{p-1}(x)$.

15. (a) Show that $\begin{cases} x = e^{4t} \\ y = e^{4t} \end{cases}$ and $\begin{cases} x = e^{-2t} \\ y = -e^{-2t} \end{cases}$ are solutions of the homogeneous system

$$\begin{cases} \frac{dx}{dt} = x + 3y \\ \frac{dy}{dt} = 3x + y \end{cases}.$$

Or

(b) Show that $\begin{cases} x = 2e^{4t} \\ y = 3e^{4t} \end{cases}$ and $\begin{cases} x = e^{-t} \\ y = -e^{-t} \end{cases}$ are solutions of the homogeneous system

$$\begin{cases} \frac{dx}{dt} = x + 2y \\ \frac{dy}{dt} = 3x + 2y \end{cases}.$$

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b)

16. (a) Show that $y = c_1 \sin x + c_2 \cos x$ is the general solution of $y'' + y = 0$ on any interval, and find the particular solution for which $y(0) = 2$ and $y'(0) = 3$.

Or

(b) Find the differential equation of $y = c_1 \sin kx + c_2 \cos kx$ and $y = c_1 + c_2 e^{-2x}$ by eliminating the constants c_1 and c_2.

17. (a) Derive Binomial Series expansion.

Or

(b) Let x_0 be an arbitrary point of the differential equation $y'' + P(x)y' + Q(x)y = 0$, and let a_0 and a_1 be arbitrary constants. Then prove that there exists a unique function $y(x)$ that is analytic at x_0, is a solution of the equation in a certain neighborhood of this point and satisfies the initial conditions $y(x_0) = a_0$ and $y'(x) = a_1$. Furthermore, prove that if the power series expansion of $P(x)$and $Q(x)$ are valid on an interval $|x - x_0| < R, R > 0$, then the power series expansion of this solution is also valid on the same interval.

18. (a) Verify that the origin is a regular singular point and calculate two independent Frobenius series solution for $2xy'' + (3-x)y' - y = 0$.

Or

(b) Prove that $\int_{-1}^{1} P_m(x)P_n(x)dx = \begin{cases} 0 & if\ m \neq n \\ \dfrac{2}{2n+1} & if\ m = n \end{cases}$, where $P_0(x), P_1(x), P_2(x), .., P_n(x)$ is asequence of orthogonal functions on the interval $-1 \leq x \leq 1$.

19. (a) Find the value of $\Gamma\left(\frac{1}{2}\right)$.

Or

(b) Prove that

$$\int_0^1 xJ_p(\lambda_m x)J_p(\lambda_n x)dx = \begin{cases} 0 & if \quad m \neq n \\ \dfrac{1}{2}J_{p+1}(\lambda_n)^2 & if \quad m = n \end{cases}$$

20. (a) Find the General solution of $\begin{cases} \dfrac{dx}{dt} = 7x + 6y \\ \dfrac{dy}{dt} = 2x + 6y \end{cases}$.

Or

(b) Find the General solution of $\begin{cases} \dfrac{dx}{dt} = -4x - y \\ \dfrac{dy}{dt} = x - 2y \end{cases}$.

(7 pages) **Reg. No. :** ....

Code No. : 6369 **Sub. Code : ZMAM 21**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

Second Semester

Mathematics – Core

ALGEBRA – II

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Let J be the ring of integers, J_n, the ring of integers modulo n. Define $\phi : J \to J_n$ by $\phi(a)$ = remainder of a on division by n. Its Kernel $I(\phi)$ is

   (a) $\{0\}$

   (b) J

   (c) the set of all multiples on n

   (d) $\{0,1,2, n-1\}$

2. Which one of the following is a maximal ideal of the ring of integers?

(a) (60) (b) (13)

(c) (2022) (d) (15)

3. A solution of the congruence $x^2 \equiv -1 (\text{mod}\, 13)$ is

(a) 3 (b) 0

(c) 5 (d) 6

4. A necessary and sufficient condition that the element a is the Euclidean ring be a unit is that

(a) $d(a) = 1$

(b) $d(a) = d(1)$

(c) $d(a) \mid d(1)$

(d) a is a prime element

5. Which one of the following is a primitive polynomial

(a) $2 + 4x^2 + 8x^5$

(b) $5 + 10x^2 + 15x^4$

(c) $20x^4 + 15x^3 + 10x^2 + 1$

(d) $2 + 2x + 2x^2$

6. Which one of the following is not true the polynomial $x^2 + 1$ is irreducible over?

(a) the complex field

(b) the real field

(c) the integers mod 3

(d) the field of rational numbers

7. In the ring of integers z, $\sqrt{(180)}$ is

(a) (4.9.5) (b) (2.3.5)

(c) (2.3.10) (d) (3.5.7)

8. In any ring R, an element $a \in rad\, R$ if and only if

(a) $1 - ra$ is invertible for each $r \in R$

(b) ra is invertible for each $r \in R$

(c) $1 - ra$ is invertible for some $r \in R$

(d) ra is invertible for some $r \in R$

9. b is a quasi-inverse of a if

(a) $ab = 1$ (b) $a + b - ab = 0$

(c) $a + b = 0$ (d) $a - b - ab = 0$

10. A ring R is isomorphic to a subdirect sum of integral domains if and only if R is

(a) semi simple

(b) simple

(c) without prime radical

(d) an integral domain

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If F is a field, prove its only ideals are (0) and F itself.

Or

(b) If ϕ is a homomorphism of R into R', prove that $\phi(0) = 0$ and $\phi(-a) = -\phi(a)$ for every $a \in R$.

12. (a) Let R be a Euclidean ring and let A be an ideal of R. Prove that there exists an elements $a_0 \in A$ such that A consists exactly of all $a_0 x$ as x ranges over R.

Or

(b) Prove that $J[i]$ is a Euclidean ring.

13. (a) If $f(x), g(x)$ are two non zero elements of $F[x]$, prove that $\deg(f(x)g(x)) = \deg f(x) + \deg g(x)$.

Or

(b) State and prove Gauss' lemma.

14. (a) Let I be an ideal of the ring R. Prove that $I \subseteq rad\, R$ if and only if each element of the coset $1 + I$ has an inverse in R.

Or

(b) For any ring R, prove that the quotient ring $R \mid Rad\, R$ is without prime radical.

15. (a) Define the J-radical $J(R)$ of a ring and a J-semi simple ring. Prove that the ring of even integers is J - semi simple.

Or

(b) Prove that a ring R is isomorphic to a sub direct sum of ring R_i, if and only if R contains a collection of ideals $\{I_i\}$ such that $R / I_i \simeq R_i$ and $\bigcap I_i = \{0\}$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If U is an ideal of the ring R, prove that $R \mid U$ is a ring and is a homomorphic image of R.

Or

(b) Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Prove that R is a field.

17. (a) Prove that the ideal $A = (a_0)$ is a maximal ideal of the Euclidean ring R is and only if a_0 is a prime element of R.

Or

(b) Let p be a prime integer and suppose that for some integer c relatively prime to p we can find integers x and y such that $x^2 + y^2 = cp$. Prove that p can be written as the sum of squares of two integers.

18. (a) Define the sum $p(x) + q(x)$ and product $p(x) \cdot q(x)$ of two polynomials and state and prove the division algorithm for polynomials.

Or

(b) State and prove the Eisenstein criterion.

19. (a) If I is an ideal of the ring R, prove that

(i) $rad\,(R/I) \supseteq \dfrac{radR + I}{I}$ and

(ii) Whenever $I \subseteq radR$, $rad(R/I) = (rad\,R)/I$

Or

(b) (i) For any ring R, prove that $radR[x] = RadR[x]$.

(ii) Let e and e' be two idempotent elements of the ring R such that $e - e' \in Rad\,R$ prove that $e = e'$.

20. (a) If R is a ring such that $J(R) \neq R$, then prove that $J(R) = \bigcap \{M \mid M$ is a modular maximal ideal of $R\}$.

Or

(b) Prove that a ring R is isomorphic to a sub direct sum of fields it and only if for each non zero ideal I of R, then exists an ideal $J \neq R$ such that $I + J = R$.

(8 pages) **Reg. No. :** ....

Code No. : 6370 **Sub. Code : ZMAM 22**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022

Second Semester

Mathematics – Core

ANALYSIS – II

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Given two partitions, P_1 and P_2. we say that P^* is their common refinement if $P^* =$ ————

   (a) $P_1 \cap P_2$ (b) $P_1 - P_2$

   (c) $P_1 \cup P_2$ (d) $P_1 + P_2$

2. $\int_{-a}^{b} f \, d\alpha$ ———— $\int_{b}^{-b} f \, d\alpha$

   (a) $\geq$ (b) $\leq$

   (c) $=$ (d) $\neq$

3. If γ is Rectifiable then $\wedge(\gamma) <$ ________

(a) 0 (b) 1

(c) 2 (d) ∞

4. If x is irrational then the value of $\lim_{m\to\infty} \lim_{n\to\infty} (\cos m!\pi x)^{2n}$ is ____________

(a) 0 (b) 1

(c) $-\infty$ (d) ∞

5. The value of $\int_0^{2\pi} (\sin n_k x - \sin n_{k+1} x)^2 \, dx =$ __________

(a) 0 (b) π

(c) 2π (d) ∞

6. If $\{f_n\}$ is uniformly bounded on E if there exists a number M such that $|f_n(x)|$ ____________M $(x \in E, n = 1,2,3,\ldots)$

(a) $<$ (b) $\leq$

(c) $>$ (d) $\geq$

7. The value of $\lim_{x\to 0} \frac{\log(1+x)}{x}$ is ____________

(a) 1 (b) 0

(c) e (d) e^x

8. The value of $\lim_{n\to\infty}\left(1+\frac{x}{n}\right)^n$ is ____________

(a) 1 (b) 0

(c) e (d) e^x

9. The value of $\int_0^\infty \left(\frac{\sin x}{x}\right)^2 dx$ is ____________

(a) $\frac{\pi}{2}$ (b) π

(c) 2π (d) $\sqrt{\pi}$

10. The value of $\Gamma\left(\frac{1}{2}\right)$ is ____________

(a) 0 (b) $\frac{1}{2}$

(c) $\sqrt{\pi}$ (d) π

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove if $f \in \mathscr{R}$ on $[a,b]$ and if there is a differentiable function F on $[a,b]$ such that $F' = f$ then $\int_a^b f(x)\,dx = F(b) - F(a)$.

Or

(b) If f is monotonic on $[a,b]$, and α is continuous on $[a,b]$, then prove that $f \in \mathscr{R}(\alpha)$.

12. (a) Suppose K is compact, and Prove that the following:

(i) $\{f_n\}$ is a sequence of continuous functions on K,

(ii) $\{f_n\}$ converges point wise to a continuous function f on K

(iii) $f_n(x) \geq f_{n+1}(x)$ for all $x \in K$, $n = 1,2,3,...$ Then $f_n \to f$ uniformly on K.

Or

(b) If f maps $[a,b]$ into R^k and if $f \in \mathscr{R}(\alpha)$ for some monotonically increasing function α *on* $[a,b]$, then Prove that, $|f| \in \mathscr{R}(\alpha)$, *and* $\left|\int_a^b f\,d\alpha\right| \leq \int_a^b |f|\,d\alpha$.

13. (a) If K is compact, if $f_n \in \mathscr{C}(\text{K})$ for $n = 1,2,3,...$, and if $\{f_n\}$ is point wise bounded and equi-continuous on K, then Prove that $\{f_n\}$ is uniformly bounded on K.

Or

(b) Let α be monotonically increasing on $[a,b]$, suppose $f_n \in \mathscr{R}(\alpha)$ *on* $[a,b]$, *for* $=1,2,3,....,$ and suppose $f_n \to f$ uniformly on $[a,b]$ Then prove that $f \in \mathscr{R}(\alpha)$ on $[a,b]$, and $\int_a^b f\,d\alpha = \lim_{n\to\infty} \int_a^b f_n d\alpha$.

14. (a) Prove that let $\mathscr{B}$ be the uniform closure of an algebra $\mathscr{A}$ of bounded functions. Then $\mathscr{B}$ is a uniformly closed algebra.

Or

(b) Suppose the series $\Sigma a_n x^n$ and $\Sigma b_n x^n$ converge in the segment $S = (-R, R)$. Let E be the set of all $x \in S$ at which $\sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} b_n x^n$, If E has a limit point in S, then $a_n = b_n$ for $n = 0,1,2,...$ Hence prove that $\sum_{n=0}^{\infty} a_n x^n = \sum_{n=0}^{\infty} b_n x^n$ holds for all $x \in S$.

15. (a) If for some x, there are constants $\delta > 0$ and $M < \infty$ such $|f(x+t) - f(x)| \le M|t|$ for all $t \in (-\delta, \delta)$, the prove that $\lim_{N\to\infty} S_N(f;x) = f(x)$.

Or

(b) Prove that if $x > 0$ and $y > 0$, then $\int_0^1 t^{x-1}(1-t)^{y-1} dt = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$ this integral is the so-called beta function $\beta(x, y)$

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Assume α increases monotonically and $\alpha' \in \mathscr{R}$ on $[a,b]$. let f be a bounded real function on $[a,b]$. Then prove $f \in \mathscr{R}(\alpha)$ iff $f\alpha' \in \mathscr{R}$ in that case $\int_a^b f\,d\alpha = \int_a^b f(x)\alpha'(x)\,dx$.

Or

(b) Prove that $f \in \mathscr{R}(\alpha)$ *on* $[a,b]$ iff for every $\varepsilon > 0$ there exists a partition P such that $U(P,f,\alpha) - L(P,f,\alpha) < \varepsilon$.

17. (a) State and Cauchy criterion for uniform convergence.

Or

(b) Prove that, If γ' is continuous on $[a,b]$, then γ is rectifiable, and $\wedge(\gamma) = \int_a^b |\gamma'(t)|\,dt$.

18. (a) Suppose $\{f_n\}$ is a sequence of functions, differentiable on $[a,b]$ and such that $\{f_n(x_0)\}$ converges for some point x_0 on $[a,b]$. if $\{f'n\}$ converges uniformly on $[a,b]$, then prove that $\{f_n\}$ converges uniformly on $[a,b]$, to function f and $f'(x) = \lim_{n\to\infty} f'n(x)$ $(a \le x \le b)$.

Or

(b) If $\{f_n\}$ is a point wise bounded sequence of complex functions on a countable set E, then Prove that $\{f_n\}$ has a subsequence $\{f_{n_k}\}$ such that $\{f_{n_k}(x)\}$ converges for every $x \in E$.

19. (a) If f is a continuous complex function on $[a,b]$, then prove that there exists a sequence of polynomials P_n such that $\lim_{n\to\infty} P_n(x) = f(x)$ uniformly on $[a,b]$, if f is real, the P_n may be taken real.

Or

(b) Suppose the series $\sum_{n=0}^{\infty} c_n x^n$ converges for $|x| < R$, and define $f(x) = \sum_{n=0}^{\infty} c_n x^n \quad (|x| < R)$. Then the prove that $\sum_{n=0}^{\infty} c_n x^n$ converges uniformly on $[-R+\varepsilon, R-\varepsilon]$, no matter which $\varepsilon > 0$ is chosen. The function f is continuous and differentiable in $(-R,R)$ and $f'(x) = \sum_{n=1}^{\infty} n c_n x^{n-1} (|x| < R)$.

20. (a) Suppose $a_0, \ldots, a_n$ are complex numbers, $n \geq 1, a_n \neq 0, P(z) = \sum_0^n a_k z^k$. Then prove that $P(z) = 0$ for some complex number z.

Or

(b) If f is a positive function on $(0,\infty)$ then Prove the following:

(i) $f(x+1) = xf(x)$

(ii) $f(1) = 1$

(iii) $\log f$ is convex, then $f(x) = \Gamma(x)$.

Reg. No. : ....

Code No. : 6371 Sub. Code : ZMAM 23

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

Second Semester

Mathematics — Core

ADVANCED CALCULUS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer.

1. Let D be the set of points (x, y) with $0 \leq x \leq 1$, $0 \leq y \leq 1$ and both x and y rational. Then

   (a) $\underline{A}(D) = \overline{A}(D) = 0$

   (b) $\underline{A}(D) = 1, \overline{A}(D) = 0$

   (c) $\underline{A}(D) = 0, \overline{A}(D) = 1$

   (d) $\underline{A}(D) = \overline{A}(D) = 1$

2. Let D be the region between the line $y = x$ and the parabola $y = x^2$. Take $f(x, y) = xy^2$. Then $\iint_D f$ is

(a) $\frac{1}{40}$ (b) $\frac{1}{20}$

(c) $\frac{1}{80}$ (d) $\frac{1}{10}$

3. Let $S : \begin{cases} u = x + y \\ v = x - y \\ w = x^2 \end{cases}$ the image of $(1, 2)$ under S is

(a) (3, 1, 1) (b) (1, 1, 0)

(c) (3, –1, 1) (d) (3, –1)

4. Let $T : \begin{cases} u = x^2 + y - z \\ v = xyz^2 \\ w = 2xy - y^2 z \end{cases}$ Then $dT\,|_{(1,1,1)}$ is

(a) $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & 2 \\ 2 & 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 1 & -1 \\ 2 & 0 & -1 \\ 1 & 1 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 2 & -1 & 1 \\ 1 & 1 & 2 \\ 2 & 0 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$

5. The Jacobian of the transformation

$$T : \begin{cases} u = x \cos y \\ v = x \sin y \end{cases} \text{ is}$$

(a) $x^2 \sin y$

(b) x

(c) $x \cos y$

(d) x^2

6. The sine and cosine functions are

(a) both linearly dependent and functionally dependent

(b) linearly independent and functionally dependent

(c) linearly dependent but not functionally dependent

(d) linearly independent but not functionally dependent

7. If γ is a smooth curve whose domain is the interval $[a, b]$ then $L(\gamma)$ is given by

(a) $\int_a^b \gamma'(t)\,dt$ (b) $\int_a^b \sqrt{|\gamma'(t)|\,dt}$

(c) $\int_a^b |\gamma'(t)|\,dt$ (d) $\int_a^b |\gamma'(t)|^2\,dt$

8. If γ is a curve of class C'' with arc length as the parameter, then the curvature of γ at the point corresponding to $t = c$ is

(a) $k = |\gamma''(c)|$ (b) $k = |\gamma'(c)|$

(c) $k = \gamma''(c)$ (d) $k = \gamma'(c)$

9. If ω is any differential form of class C'', then ddw is

(a) w (b) 0

(c) $-w$ (d) w^*

10. If $x = u^2 + v$, $y = v$ and $\sigma = xy^2 dxdy$ then σ^* is

(a) $(u^3v^2 + uv^3)dudv$

(b) $2uv^2dudv$

(c) $(2u^3v^2 + 2uv^3)dudv$

(d) $2uv^3dudv$

PART B — (5 × 5 = 25 marks)

Answer ALL questions choosing either (a) or (b).

11. (a) Let f and g be continuous and bounded on D prove that $\iint_D (f+g) = \iint_D f + \iint_D g$ and $\iint_D Cf = C\iint_D f$ for any constant C.

Or

(b) Show that for $x > 0$

$$\int_0^{\frac{\pi}{2}} \log[\sin^2\theta + x^2\cos^2\theta]\,d\theta = \pi\log\left(\frac{x+1}{2}\right)$$

12. (a) Consider the following linear transformations of the plane into itself.

$$S:\begin{cases} u = 2x - 3y \\ v = x + y \end{cases} \quad T:\begin{cases} u = x + y \\ v = 3x + y \end{cases}$$

Find ST and TS and check whether $ST = TS$ or not.

Or

(b) For any $P \in S$ and any $u \in R^3$, prove that $dg|_p\,(u) = Dg(p) \cdot u$, where g is a real valued function of class C' defined on an open set S in 3-space.

13. (a) Let T be a transformation from R^n into R^n which is of class C' in an open set D and suppose that $J(P) \neq 0$ for each $P \in D$. Prove that T is locally 1-to-1 in D.

Or

(b) Let F and G be of class C' in an open set $D \subset R^5$. Let $p_0 = (x_0, y_0, z_0, u_0, v_0)$ be a point of D at which both of the equations. $F(x, y, z, u, v) = 0$, $G(x, y, z, u, v) = 0$ are satisfied. Suppose also that $O(F, G)/O(u, v) \neq 0$ at p_0. Prove that there are two function ϕ and ψ of class C' in a neighborhood N of (x_0, y_0, z_0) such that $u = \phi(x, y, z)$, $v = \psi(x, y, z)$ is a solution of $F = G = O$ in N giving u_0 and v_0 at (x_0, y_0, z_0).

14. (a) If E is a closed bounded subset of Ω of zero volume, prove that $T(E)$ has zero volume.

Or

(b) Let γ_1 and γ_2 be smoothly equivalent smooth curves, and let p be a simple point on their trace. Prove that γ_1 and γ_2 have the same direction at p.

15. (a) Let $\overline{a} = 2i - 3j + k, \overline{b} = i - j + 3k$, $\overline{c} = i - 2j$. Compute the vectors $(\overline{a} \times \overline{b}) \cdot \overline{c}$ and $a \times (b \times c)$.

Or

(b) If ω is any differential form of class C'', prove that $dd\omega = 0$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If f is continuous on R, prove that $\iint_R f$ exists.

Or

(b) Let R be the rectangle described by $a \le x \le b, c \le y \le d$ and let f be continuous on R. Prove that $\iint_R f = \int_a^b dx \int_c^d f(x, y)\, dy$.

17. (a) Define a linear transformation. Let L be a linear transformation from R^n into R^m represented by the matrix $[a_{ir}]$. Prove that there is a constant B such that

$|L(P)| \le B|P|$ for all points P.

Or

(b) Let T be differentials on an open set D and let S be differentiate on an open set containing $T(D)$. Prove that ST is differentiable on D and if $P \in D$ and $q = T(P)$ then $d(ST)|_p = dS|_q \; dT|_p$.

18. (a) Let T be of class C' on an open set D in n space, taking values in n space. Suppose that $J(P) \neq r$ for all $P \in D$. Prove that $T(D)$ is an open set.

Or

(b) Prove that the local inverses are themselves differentiable transformations and find a formula for their differentials.

19. (a) If γ is a smooth curve whose domain is the interval $[a, b]$, prove that γ is rectifiable and also show that $L(\gamma) = \int_a^b |\gamma'(t)| \, dt$.

Or

(b) Let F be an additive set function, defined on $\mathcal{O}$ and a.c. suppose also that F is differentiable everywhere, and uniformly differentiable on compact sets, with the derivative a point function *f*. Prove that f is continuous everywhere and $F(s) = \iint_s f$ holds for every rectangle S.

20. (a) Let D be a closed convex region in the plane and let $w = A(x, y)\,dx + B(x, y)\,dy$ with A and B of class C' in D. Prove that $\iint\limits_{\partial D} Adx + Bdy = \iint\limits_{D} dw = \iint\limits_{D} \left(\frac{\partial B}{\partial x} - \frac{\partial A}{\partial y} \right) dxdy$.

Or

(b) If α is a k form and β any differential form, prove that $d(\alpha\beta) = (d\alpha)\,\beta + (-1)^k \alpha\,(d\beta)$.

(7 pages) **Reg. No. :** ....

Code No. : 6372 **Sub. Code : ZMAM 24**

M.Sc. (CBCS) DEGREE EXAMINATION, NOVEMBER 2022.

Second Semester

Mathematics – Core

DIFFERENTIAL GEOMETRY

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The arc length from a to any point u is given by

(a) $r = S(u) = \int_a^u R(u) du$

(b) $r = S(u) = \int_a^u |\dot{R}(u)| du$

(c) $r = S(u) = \int_a^u |R(u)| du$

(d) $r = S(u) = \int_a^u |\dot{R}(u)|^2 du$

2. The line of intersection of the nominal plant and the osculating plane at P is called

(a) the principal normal

(b) the principal tangent

(c) the principal curvature

(d) the principal line

3. The radius of spherical curvature is given by

(a) $R = \sqrt{\rho^2 + \sigma^2\rho^2}$

(b) $R = \sqrt{\rho^2 + \rho'^2}$

(c) $R = \sqrt{\rho^2 + \sigma^2\rho'^2}$

(d) $R = \sqrt{\rho^2 + \sigma^2}$

4. The equation of the involute is

(a) $R = r + t$

(b) $R = r + \rho n + \sigma\rho' b$

(c) $[R - r, \dot{r}, \ddot{r}] = 0$

(d) $R = r + (c - s)t$

5. For the paraboloid $x = u$, $y = v$, $z = u^2 - v^2$, F is

(a) $1 + 4u^2$

(b) $1 + 4v^2$

(c) $4uv$

(d) $-4uv$

6. The two parametric curves through a point P are orthogonal if at P

(a) $r_1 \times r_2 = 0$

(b) $r_1 \cdot r_2 = 0$

(c) $r_1 + r_2 = 0$

(d) $r_1 \times r_2 \neq 0$

7. When $v = c$ for all values of u, a necessary and sufficient condition that the curve $v = c$ is a geodesic is

(a) $EE_2 + FE_1 + 2EF_1 = 0$

(b) $EE_2 + FE_1 - 2EF_1 = 0$

(c) $GC_1 + FG_2 - 2GF_2 = 0$

(d) $EE_2 - FE_1 + 2EF_1 = 0$

8. A necessary and sufficient condition for a curve $u = u(t)$, $v = v(t)$ on a surface $r = r(u,v)$ to be geodesic is that

(a) $U\frac{\partial T}{\partial \dot{v}} + V\frac{\partial T}{\partial \dot{u}} = 0$

(b) $U\frac{\partial T}{\partial v} - V\frac{\partial T}{\partial u} = 0$

(c) $U\frac{\partial T}{\partial \dot{v}} - V\frac{\partial T}{\partial \dot{u}} = 0$

(d) $U - V\frac{\partial T}{\partial \dot{u}} = 0$

9. The second fundamental form is

(a) $Edu^2 + 2Fdudv + Gdv^2$

(b) $Ldu^2 + 2Mdudv + Ndv^2$

(c) $Lh^2 + Mhk + N$

(d) $Ldv^2 + 2Mdu + Ndu^2$

10. The Gaussian Curvature K is defined by

(a) $K = \frac{1}{2}(K_a + K_b)$ (b) $K = K_a K_b$

(c) $K = \frac{1}{2}\sqrt{K_a K_b}$ (d) $K = LN - M^2$

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $[r', r'', r'''] = \kappa^2 \tau$.

Or

(b) Find the curvature and the torsion of the curve given by $r = \{a(3u - u^3), 3au^2, a(3u + u^3)\}$.

12. (a) Prove that the osculating plane at any point P has three point contact with the curve at P.

Or

(b) Prove that the projection C_1 of a general helix C on a plane perpendicular to its axis has its principal normal parallel to the corresponding principal normal of the helix and its corresponding curvature is given be $k = k_1 \sin^2 \alpha$.

13. (a) Show that the matrix is a positive definite quadratic form in du, dv.

Or

(b) Find E,F,G and H for the paraboloid

$x = y, y = v, z = u^2 - v^2$

14. (a) Prove that, on the general surface, a necessary and sufficient condition that the curve $v = c$ be a geodesic is $EE_2 + FE_1 - 2EF_1 = 0$ when $v = c$ for all values of u.

Or

(b) On the paraboloid $x^2 - y^2 = z$ find the orthogonal trajectories of the sections by the planes z = constant.

15. (a) Prove that if the orthogonal trajectories of the curves v = constant are geodesics, then H^2 / E is independent of u.

Or

(b) If K is the normal curvature in a direction making an angle ψ write the principle direction v = constant then prove that $K = K_a \cos^2 \psi + K_b \sin^2 \psi$.

PART C — $(5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) State and prove Serret Frenet formulae.

Or

(b) Show that the length of the common perpendicular d of the tangents at two near points distance s apart is approximately given by $d = \frac{k\tau s^3}{12}$.

17. (a) Define the osculating sphere and the centre A spherical curvature. If a curve lies on a sphere show that ρ and σ are related by $\frac{d}{ds}(\sigma\rho') + \frac{\rho}{\sigma} = 0$.

Or

(b) Prove that $\frac{K}{\tau} =$ constant is a characteristic property of helices.

18. (a) Find E, F, G and H for the anchor ring and find the area of the anchor ring corresponding to the domain $0 \le u \le 2\pi$ and $0 \le v \le 2\pi$.

Or

(b) If (l', m') are the direction coefficients of a line which makes an angle $\pi/2$ with the line whose direction coefficients are (l, m), then prove that $l' = -\frac{1}{H}(Fl + Gm), m' = \frac{1}{H}(El + Fm)$.

19. (a) Prove that every helix on an cylinder is a geodesic.

Or

(b) Prove that any curve $u = u(t), v = v(t)$ on a surface $r = r(u, v)$ is a geodesic if and only if the principal normal at every point on the curve is normal to the surface.

20. (a) State and prove Liouville's formula for Kg.

Or

(b) Prove that a necessary and sufficient condition for a curve on a surface to be a line of curvature is $kdr + dN = 0$ at each point on the line of curvature where K is the normal curvature in the direction dr of the line of curvature.

(7 pages) **Reg. No. :** ....

Code No. : 6373 **Sub. Code : ZMAM 25**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022

Second Semester

Mathematics – Core

RESEARCH METHODOLOGY AND STATISTICS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. The one person who will almost certainly feature in the acknowledgement is

   (a) your father (b) your supervisor

   (c) your principal (d) your friend

2. Typically, abstracts are between ———— and ———— words in length.

   (a) 250 and 300 (b) 25 and 30

   (c) 2020 and 3000 (d) 5 and 10

3. Suppose a box contains 3 white balls and 2 black balls two balls are to be drawn successively at random and without replacement the probability that both balls drawn are black in

(a) $\frac{1}{5}$ (b) $\frac{1}{10}$

(c) $\frac{2}{5}$ (d) $\frac{1}{4}$

4. Let the joint p.d.f of X_1 and X_2 be

$$f(x_1, x_2) = \begin{cases} x_1 + x_2, & 0 < x_1 < 1, 0 < x_2 < 1 \\ 0 & elsewhere \end{cases}$$

The marginal probability density function $f(x_1)$ in $0 < x_1 < 1$ is

(a) $x_1 + x_2$ (b) $x_1 + 1$

(c) $x_1 \times x_2$ (d) x_1

5. If X has the p.d.f $f(x) = \begin{cases} \frac{1}{4} x e^{-x/2} & 0 < x < \infty \\ 0 & elsewhere \end{cases}$, then X is

(a) $\chi^2(2)$ (b) $\chi^2(8)$

(c) $\chi^2(-\frac{1}{2})$ (d) $\chi^2(4)$

6. If $M(t) = \rho^{3t+8t^2}$, then σ is

(a) 4 (b) 8

(c) 3 (d) 16

7. If $x_1 = \frac{1}{2}(y_1 + y_2)$, $x_2 = \frac{1}{2}(y_1 - y_2)$ then the value of J is

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$

(c) 2 (d) –2

8. If U and V are stochastically independent chi-square variables with r_1 and r_2 degrees of freedom, then which one of the following is an F-distribution

(a) $\frac{Ur_1}{Vr_2}$ (b) $\frac{U/r_1}{V/r_2}$

(c) $\sqrt{\frac{U/r_1}{V/r_2}}$ (d) $\frac{U/r_2}{V/r_1}$

9. Let X_1 and X_2 be stochastically independent with normal distribution $n(6,1)$ and $n(7,1)$ respectively, then $X_1 - X_2$ is

(a) $n(-1,2)$ (b) $n(1,2)$

(c) $n(-1,1)$ (d) $n(0,1)$

10. Let $\overline{X}$ denote the mean of a random sample of size 128 from a gamma distribution with $\alpha = 2$ and $s = 4$ the variance of $\overline{X}$ is

(a) $\frac{8}{128}$ (b) $\frac{3}{128}$

(c) $\frac{1}{2}$ (d) $\frac{1}{4}$

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Write a model Acknowledgement for your research project.

Or

(b) Why do we have a literature review?

12. (a) Show that the random variables X_1 and X_2 with joint p.d.f $f(x_1, x_2) = 12x_1x_2(1 - x_2)$, $0 < x_1 < 1, 0 < x_2 < 1$, zero elsewhere, are stochastically independent.

Or

(b) Two dimensional random variable (x, y) has the joint p.d.f. $f(x, y) = 8xy, 0 < x < y < 1$, zero elsewhere. Find marginal and conditional distributions.

13. (a) Find the m.g.f of a gamma distribution.

Or

(b) If the random variable x is $N(\mu,\sigma^2)$ $\sigma^2 > 0$, prove that the random variable $W = (x-\mu)/\sigma$ is $N(0,1)$.

14. (a) Show that $S^2 = \frac{1}{n}\sum_1^n (x_i - \overline{x})^2 = \frac{1}{n}\sum_1^n x_1^2 - \overline{x}^2$ what $\overline{X} = \sum_1^n X_i / n$.

Or

(b) Let X have the p.d.f. $f(x)=1$, $0 < x < 1$ zero elsewhere. Show that the random variable $y = -2\log x$ has a chi-square distribution with z degrees of freedom.

15. (a) If $X_1, X_2, \ldots X_n$ is a random sample from a distribution with m.g.f $M(t)$, show that the m.g.f of $\sum_1^n x_i$ and $\sum_1^n X_i / n$ are respectively, $[M(t)]^n$ and $[M(t/n)]^n$.

Or

(b) Let $\overline{X}$ denote the mean of a random sample of size 100 from a distribution test is $\chi^2(50)$. Compute an approximare value of $\Pr(49 < \overline{X} < 51)$.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) What are the different components of a research project? Explain.

Or

(b) (i) Explain how referencing convention are applied according to their sources.

(ii) Explain plagiarism. How will you avoid it in your project report.

17. (a) Let X_1 and X_2 have joint p.d.f $f(x_1, x_2) = 2, 0 < x_1 < x_2 < 1$ zero elsewhere

Find (i) the marginal probability density function

(ii) Conditional p.d.f of x_1 given $x_2 = x_2, 0 < x_2 < 1$

(iii) Condition mean and conditional variance of x_1 given $x_1 = x_2$.

Or

(b) Let $F(x_1, x_2) = 2(x_1^2 x_2^3, 0 < x_1 < x_2 < 1$, zero elsewhere, be the joint p.d.f pg x_1 and x_2 find the conditional mean and variance of x_1. given $x_2 = x_2$, $0 < x_2 < 1$.

18. (a) Find the m.g.f of the normal distribution and hence find the mean and variance of a normal distribution

Or

(b) If the random variable X is $N(\mu,\sigma^2),\sigma^2 > 0$, prove that the random variable $V = (x - \mu^2 / \sigma^2 \, is \, \chi^2(1)$.

19. (a) Let X_1, X_2 be a random sample of size $n = 2$ from a standard normal distribution. Show that the marginal p.d.f of $y_1 = x_1 / x_2$ is that of a Cauchy distribution. You mat take $y_2 = x_2$.

Or

(b) Derive the F-distribution.

20. (a) Let $X_1, X_2, \ldots X_n$ denote a random sample of size $n \geq 2$ from a distribution that is $N(\mu,\sigma^2)$. Let S^2 be the variance of this random sample prove that $\frac{nS^2}{\sigma^2}$ is a $\chi^{2`}$-variable with parameter $n-1$.

Or

(b) State and prove a special case of the central limit theorem.

(8 pages) **Reg. No. :** ....

Code No. : 6374 **Sub. Code : ZMAE 21**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

Second Semester

Mathematics

Elective — CLASSICAL MECHANICS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. The equation of motion is a differential equation of order

   (a) 3 (b) 0

   (c) 1 (d) 2

2. If there exist holonomic constrains expressed in 5 equations, then the system of 50 particles has ———— degrees of freedom.

(a) 55 (b) 45

(c) 145 (d) 150

3. The Lagrangian L is defined as

(a) $T + V$

(b) $T - V$

(c) $V - T$

(d) $T.V$

4. The equation $\sum_i F_i^{(a)} \cdot \delta r_i = 0$ is called

(a) the principle of virtual work

(b) the equation of motion

(c) the Lagrangian equation

(d) the Maxwell equation

5. The total length of any curve going between points 1 and 2 is

(a) $\int_{x_1}^{x_2} \left(1 + \left(\frac{dy}{dx} \right)^2 \right) dx$

(b) $\int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx} \right)^2} \cdot dx$

(c) $\int_{x_1}^{x_2} \left(1 + \left(\frac{dy}{dx} \right)^{1/2} \right) dx$

(d) $\int_{x_1}^{x_2} \sqrt{\left(\frac{dy}{dx} \right) + \left(\frac{dy}{dx} \right)^2}\ dx$

6. For any parametric family of curves $J(\alpha) = \int_{x_1}^{x_2} f\,(y(x, \alpha), \dot{y}\,(x, \alpha), x)\, dx$ the condition for obtaining a stationary point is

(a) $\left. \left(\frac{dJ}{dx} \right)_{\alpha=0} \right| = 0$

(b) $\left. \left(\frac{dJ}{d\alpha} \right)_{\alpha=0} \right| = 0$

(c) $\left. \left(\frac{dJ}{d\alpha} \right)_{x=0} \right| = 0$

(d) $\left. \left(\frac{dJ}{d\alpha} \right)_{y=0} \right| = 0$

7. The areal velocity is

(a) $n^2\dot{\theta}$ (b) $\frac{1}{2}r^2\,\theta$

(c) $\frac{1}{2}r^2\,\dot{\theta}$ (d) $r^2\,\dot{\theta}$

8. Consider a plot of V' against r for the specific case of an attractive inverse square law of force $f=-\frac{k}{r^2}$. The potential energy for this force is

(a) $V=\frac{k}{r}$ (b) $V=\frac{k}{r^2}$

(c) $V=-\frac{k}{r}$ (d) $V=0$

9. The eccentric anomaly ψ is defined by the relation

(a) $r=a\,(1+e\cos\psi)$ (b) $r=a\,(1-e\cos\psi)$

(c) $r=a\,(1-\cos\psi)$ (d) $\psi=\frac{ae}{r^2}$

10. The Kepler's equation is

(a) $\omega\,t=\psi=e\,\sin\psi$ (b) $\omega\,t=\psi+e\,\sin\psi$

(c) $\omega\,t=\psi=e\,\cos\psi$ (d) $\omega\,t=\psi+e\,\cos\psi$

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove the conservation theorem for the angular momentum of a particle.

Or

(b) Define a rigid body and show that in a rigid body the internal forces do no work. What can you say about the internal potential of such body.

12. (a) Obtain the Lagrange equations of motion for a spherical pendulum.

Or

(b) Derive the Lagrange equation of motion of a bead sliding on a uniformly rotating wire in a force-free space.

13. (a) Show that the shortest distance between two points in a plane is a straight line.

Or

(b) Explain Braschistochrone problem.

14. (a) Show that the central force motion of two bodies about their center of mass can always be reduced to an equivalent one-body problem.

Or

(b) Two particles move about each other in circular orbits under the influence of gravitational forces, with a period τ. Their motion is suddenly stopped at a given instant of time, and they are then released and allowed to fall into each other. Prove that they collide after a time $\tau/4\sqrt{2}$

15. (a) Obtain the differential equation for the orbit if the force law f is known.

Or

(b) Prove that for the Kepler problem there exists a conserved vector A defined by $\overline{A} = \overline{P} \times \overline{L} - m\,k\frac{\overline{r}}{r}$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions choosing either (a) or (b).

16. (a) State and prove the conservation theorem for the linear momentum of a system of particles.

Or

(b) (i) Explain holonomic and nonholonomic constraints with suitable examples.

(ii) State the two types of difficulties due to constraints in the solution of mechanical problems.

17. (a) Derive Lagrange's equation of motion from D'Alembert's principle.

Or

(b) Show that the kinetic energy of a system can always be written as the sum of three homogeneous functions of the generalized velocities.

18. (a) Derive the Euler-Lagrange differential equations.

Or

(b) Derive Lagrange's equations for non holonomic systems.

19. (a) State and prove Kepler's second law of planetary motion.

Or

(b) A particle moves in a central force field given by the potential $V = -k\frac{e^{-ar}}{r}$ where k and a are positive constant. Using the method of the equivalent one-dimensional potential discuss the nature of the motion.

20. (a) Obtain the equation of motion for the particle moving under the influence of a central force $f = -k/r^2$.

Or

(b) (i) For the Kepler's equation $\omega t = \psi - e \sin \psi$ prove that $\tan \theta/2 = \sqrt{\frac{1+e}{1-e}} \tan \psi/2$

(ii) Derive the orbit equation for the Kepler problem using Laplace–Runge–Lenz vector.

(7 pages) **Reg. No. :** ....

Code No. : 6375 **Sub. Code : ZMAE 22**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

Second Semester

Mathematics — Core

Elective — PARTIAL DIFFERENTIAL EQUATIONS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. The Pfaffian differential equation $X.dr = 0$ is integrable if and only if

   (a) $X.curl\ x = 0$ (b) $X.dr = 0$

   (c) curl grad $X = 0$ (d) $\nabla^2 X = 0$

2. The solution of the equation $a\,y^2\,z^2dx + b\,z^2x^2dy + c\,x^2y^2dz = 0$ is ____________ (where k is a constant).

(a) $ax + by + cz = k$

(b) $\frac{a}{x^2} + \frac{b}{x^2} + \frac{c}{z^2} = k$

(c) $\frac{a^2}{x} + \frac{b^2}{y} + \frac{c^2}{z} = k$

(d) $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = k$

3. The set of all spheres with centers on the z axis is characterized by the partial differential equation

(a) $yp - xq = 0$ (b) $yp + xq = 0$

(c) $yq - xp = 0$ (d) $z^2\left(1 + p^2 + q^2\right) = 1$

4. Eliminate the constants a and b from $z = (x + a)(y + b)$

(a) $px + qy = 0$ (b) $z = ab$

(c) $z = \frac{p}{q}$ (d) $z = pq$

5. If $f = xp - yq$, $g = z(xp + yq) - 2xy$ then $\dfrac{\partial (f, g)}{\partial (x, p)}$ is

(a) $-x^2 p$ (b) $2xy$

(c) $-2xy$ (d) 0

6. $[f, g]$ means

(a) $\dfrac{\partial (f, g)}{\partial (x, p)} + p\dfrac{\partial (f, g)}{\partial (z, p)} + \dfrac{\partial (f, g)}{\partial (y, q)} + q\dfrac{\partial (f, g)}{\partial (z, q)}$

(b) $\dfrac{\partial (f, g)}{\partial (x, p)} + \dfrac{\partial (f, g)}{\partial (z, p)} + \dfrac{\partial (f, g)}{\partial (y, q)} + \dfrac{\partial (f, g)}{\partial (z, q)}$

(c) $\dfrac{\partial (f, g)}{\partial (x, p)} + \dfrac{\partial (f, g)}{\partial (y, q)}$

(d) $\dfrac{\partial (f, g)}{\partial (x, p)} + q\dfrac{\partial (f, g)}{\partial (z, p)} + \dfrac{\partial (f, g)}{\partial (y, q)} + p\dfrac{\partial (f, g)}{\partial (z, q)}$

7. $\dfrac{\partial^2 \phi}{\partial x^2} = \dfrac{1}{k}\dfrac{\partial \phi}{\partial t}$ is known as

(a) One-dimensional wave equation

(b) One-dimensional diffusion equation

(c) Poisson's equation

(d) One-dimensional differential equation

8. $F(D, D')\{e^{ax+by}\phi(x, y)\}$ is

(a) $e^{ax+by} F(D+a, D'+b)\phi'(x,y)$

(b) $F(a,b)\phi(x,y)$

(c) $e^{ax+by}F(D+a, D'+b)\phi(x, y)$

(d) $e^{ax+by} F(a,b)\phi(x, y)$

9. If the determinant $\begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix}$ of the form ϕ vanishes, we say that the equation is

(a) hyperbolic (b) elliptic

(c) parabolic (d) circle

10. Classify the equation $U_{xx} + U_{yy} = U_z$

(a) elliptic (b) parabolic

(c) hyperbolic (d) circle

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the integral curves of the equations

$$\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$$

Or

(b) Verify that the equation $x\left(y^2-a^2\right)dx+y\left(x^2-z^2\right)dy-z\left(y^2-a^2\right)dz=0$ is integrable and solve it.

12. (a) Find the integral surface of the linear partial differential equation

$x\left(y^2+z\right)p-y\left(x^2+z\right)q=\left(x^2-y^2\right)z$ which contains the straight line $x+y=0, z=1$.

Or

(b) Find the general integral of the linear partial differential equation

$y^2p-xyq=x\left(z-2y\right)$

13. (a) Show that the equation $z=px+zy$ is compatible with any equation $f\left(x, y, z, p, q\right)=0$ which is homogeneous in x, y, z.

Or

(b) Find a complete integral of the equation $p^2x+q^2z=z$.

14. (a) Find a particular integral of the equation $\left(D^2-D'\right)z=e^{x+y}$.

Or

(b) Solve the equation $r+s-2t=e^{x+y}$.

15. (a) Classify the equation $U_{xx} + U_{yy} = U_{zz}$.

Or

(b) Find a solution of the form $z = X(x)Y(y)T(t)$ of the equation

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{1}{k} \cdot \frac{\partial z}{\partial t}$$

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions choosing either (a) or (b).

16. (a) Find the integral curves of the equations

$$\frac{a\,dx}{(b-c)\,yz} = \frac{bdy}{(c-a)\,zx} = \frac{cdz}{(a-b)\,xy}$$

Or

(b) Solve the equation $(x^2 z - y^3)\,dx + 3xy^2 dy + x^3 dz = 0$ first showing that is integrable.

17. (a) If U is a function of x, y and z which satisfies the partial differential equation

$$(y-z)\frac{\partial u}{\partial x} + (z-x)\frac{\partial u}{\partial y} + (x-y)\frac{\partial u}{\partial y} = 0,$$ show that U contains x, y and z only in combinations $x+y+z$ and $x^2+y^2+z^2$.

Or

(b) Prove that a necessary and sufficient condition that there exists between $U(x, y)$ and $V(x, y)$ a relation $F(u, v) = 0$ not involving x or y explicitly is that $\frac{\partial(u, v)}{\partial(x, y)} = 0$.

18. (a) Find the solution of the equation $z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y)$ which passes through the x-axis.

Or

(b) Show that the equations $xp - yq = x$ and $x^2 p + q = xz$ are compatible and find their solution.

19. (a) Derive the telegraphy equation.

Or

(b) Find the solution of the equation

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$$

20. (a) Solve the one-dimensional diffusion equation

$$\frac{\partial^2 z}{\partial x^2} = \frac{1}{k}\frac{\partial z}{\partial t}$$

Or

(b) Solve the one-dimensional wave equation

$$\frac{\partial^2 z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 z}{\partial t^2}$$

Reg. No. : ....

Code No. : 6376 **Sub. Code : ZMAE 23**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

Second Semester

Mathematics

Elective — PYTHON PROGRAMMING – THEORY

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. ————————— is a single action in computer

(a) selection (b) statement

(c) sequence (d) iteration

2. All instructions are executed one after another is called —————————

(a) sequence (b) statement

(c) selection (d) iteration

3. ———— is the program construct that allows program to choose between different actions

(a) selection (b) looping

(c) iteration (d) sequence

4. ———— is a collection of well defined and computable instructions to execute and get the proper output

(a) algorithm (b) selection

(c) iteration (d) pseudocode

5. ———— statement displays a result

(a) display (b) printf

(c) print (d) output

6. ———— is not a keyword in python

(a) lambda (b) try

(c) from (d) to

7. ———— is used to refer to a value passed as an argument inside function

(a) body (b) header

(c) parameter (d) object

8. ———— function always returns none

(a) void (b) fruitful

(c) local (d) global

9. ———— is a sequence of values

(a) string (b) list

(c) tuple (d) none

10. ———— is a part of string specified by a range of indices

(a) index (b) traverse

(c) immutable (d) slice

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Explain in detail about algorithm quality.

Or

(b) Discuss about functions

12. (a) Explain in detail about pseudocode for mathematical problems.

Or

(b) Write an algorithm to calculate factorial numbers.

13. (a) Explain in detail about assignment statements in python.

Or

(b) Explain briefly about operators in python.

14. (a) Explain briefly about fruitful functions.

Or

(b) Explain briefly about function composition.

15. (a) Explain briefly about list operations.

Or

(b) Explain briefly that Tuples are immutable.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b)

16. (a) Discuss in detail about problem solving techniques.

Or

(b) Discuss the building blocks of algorithms.

17. (a) Explain in detail about strategies for developing algorithms.

Or

(b) Write an algorithm to find the greatest of two numbers.

18. (a) Explain in detail about operators in python.

Or

(b) Explain in detail about values and types in python.

19. (a) Explain in detail about parameters and arguments.

Or

(b) Explain briefly about while and for statement.

20. (a) Explain in detail about string functions.

Or

(b) Discuss about lists mutability.

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(8 pages)

Reg. No. : ....

Code No. : 6377 **Sub. Code : ZMAM 31**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

Third Semester

Mathematics — Core

ADVANCED ALGEBRA — I

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. If dim V=5 then dim Hom (V, V) + dim Hom (V, F) is

(a) 50 (b) 25

(c) 10 (d) 30

2. An R-module M is said to be cyclic if there is an element $m_0 \in m$ such that every $m \in M$ is of the form

(a) $m = rm_o$ for some $r \in R$

(b) $m = m_o{}^n$ for some integer n

(c) $m = r + m_o$ for some $r \in R$

(d) $m = rm_o$ for some $n \in M$

3. If V is finite dimensional over F and if $T \in A(V)$ is singular, then there exists an $S \neq 0$ in A(V) such that

(a) ST=TS=1

(b) ST=TS=0

(c) $vS = 0$ for some $v \neq 0$ in V

(d) ST=TS

4. If $vT = \lambda v$ the vT^k is

(a) $(\lambda v)^k$

(b) λv^k

(c) $\lambda^k v^k$

(d) $\lambda^k v$

5. If M, of dimension m, is cyclic w.r.t. T, then the dimension of MT^k is

(a) $\frac{m}{k}$ (b) $m+k$

(c) $m-k$ (d) m^k

6. Which one of the following is a Jordan block

(a) $\begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 3 & 0 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 3 & 1 & 0 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 3 \end{pmatrix}$ (d) $\begin{pmatrix} 3 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 3 \end{pmatrix}$

7. $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -3 \end{pmatrix}$ is the companion matrix of

(a) $1+3x+3x^2$

(b) $1+3x+3x^2+x^3$

(c) $-1-3x-3x^2-x^3$

(d) $-1-3x-3x^2+x^3+x^4$

8. Which one of the following is not true for all $A, B \in Fn$

(a) $(A')' = A$

(b) $(A+B)' = B' + A'$

(c) $(AB)' = A'B'$

(d) $(\lambda A')' = \lambda A'$ where $\lambda \in F$

9. The normal transformation N is unitary if and only if its characteristic roots are

(a) real

(b) complex number

(c) all of absolute value 1

(d) all equal to 1

10. The signature of the real quadratic form $x_1^{\ 2} + 2x_1x_2 + x^2$ is

(a) 0

(b) –1

(c) 1

(d) 2

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Define Ham(V, W). Introduce an addition and a scalar multiplication in Hom (V, W). Show that if $S, T \in Hom(V, W)$ than $S + T \in Hom(V, W)$.

Or

(b) Let V be the set of all continuous complex - valued functions on [0, 1]. If $f(t), g(t) \in V$, define $(f|t|, g|t|) = \int f(t)\overline{g|t|}\,dt$. Prove that this defines an inner product on V.

12. (a) If $\lambda \in F$ is a characteristic root of $T \in A(V)$. Prove that for any polynomial of $q(x) \in F(x)$, $q(\lambda)$ is a characteristic root of q(T).

Or

(b) Let V be the vector space of all polynomials over F of degree 3 or less and let D be the differentiation operator defined on V. Find the matrix of D w.r.t. the basis

(i) $1, x, x^2, x^3$

(ii) $1, 1+x, 1+x^2, 1+x^3$

13. (a) If $W \subset V$ is invariant under T, prove that T induces a linear transformation $\overline{T}$ on V/W defined by $(V+W)\overline{T} = vT + w$.

Or

(b) If $T \in A(V)$ is nilpotent, prove that $\alpha_0 + \alpha_1 + T + \ldots + \alpha m T^m$, where the $\alpha_i \in F$, is invertible if $\alpha_o \neq 0$.

14. (a) If V is cyclic relative to T and if the minimal polynonomial of T in F[x] is $p(x)$, then prove that for some basis of V, the matrix of T is C(p(x)), the companion matrix of $p(x)$.

Or

(b) If A is invarible, prove that det $A \neq 0$, det $(A^{-1})=(\det A)^{-1}$ and det $(ABA^{-1})=\det B$ for all B.

15. (a) If $T \in A(V)$, prove that $T^{*} \in A(V)$ and $\left(T^{*}\right)^{*} = T$.

Or

(b) Let N be a normal transformation and suppose that λ and μ are two distinct characteristic roots of N. If V, W are in V are such that $VN = \lambda v,\ wN = \mu w$ prove that $(v, w) = 0$.

PART C — $(5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) If V and W are of dimensions m and n respectively exhibit a basis of Han $(v,\ w)$ over F consisting of mn elements.

Or

(b) Let R be a Euclidean ring. Prove that any finitely generated R-module M is the direct sum of a finite number of cyclic submodules.

17. (a) For arbitrary algebras A with unit element over a field F, state and prove the analog of cayley's theorem for groups.

Or

(b) What relation, if any, must exist between characteristic vectors of T belonging to different characteristic roots? Explain your answer.

18. (a) If V is n-dimensional over F and if $T \in A(V)$ has all its characteristic roots in F, prove that T statisfies a polynomial of degree n over F.

Or

(b) Let $T \in A(v)$ and suppose that $p(x) = q_1(x)^{l1} q_2(x)^{l2} \ldots q_k(n)^{lk}$ in $F(x)$ is the minimal polynomial of T over F. For each $i = 1,2\ldots\ldots k$, define $V_i = \{v \in V \mid (V_{qi}(T))^{li} = 0\}$ prove that $v_i \neq 0$ for each i and $V = V_1 \oplus V_2 \oplus \ldots \oplus Vk$.

19. (a) Prove that the elements S and T in A(V) are similar in A(V) if and only if they have the same elementary divisions.

Or

(b) For $A, B \in Fn$ and $\lambda \in F$, Prove that

(i) $tr(\lambda A) = \lambda t_r A$

(ii) $tr(A + B) = trA + trB$.

(iii) $tr(AB) = tr(BA)$.

20. (a) Prove that to linear transformation T on V is unitary if and only if it takes an orthonomal basis of V into an orthonormal basis of V.

Or

(b) State and prove that sylvester's law of inertia.

(6 pages) **Reg. No. :** ....

Code No. : 6378 **Sub. Code : ZMAM 32**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

Third Semester

Mathematics

GRAPH THEORY

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. The number of edges in $K_{4,5}$ is

(a) 9 (b) 20

(c) 1 (d) 45

2. Consider the graph G : 

In $G-e$, the number of vertices is

(a) 2 (b) 9

(c) 6 (d) 8

3. If G is a tree with 16 edges then the number of vertices of G is

(a) 17

(b) 15

(c) 0

(d) any number between 1 and 16

4. Which one of the following is nit true?

It e is link of G then

(a) $\gamma(G \cdot e) = \gamma(G) - 1$

(b) $\varepsilon(G \cdot e) = \varepsilon(G) - 1$

(c) $\omega(G \cdot e) = \omega(G) - 1$

(d) $G \cdot e$ is a tree is G is a tree

5. In the Konigsberg bridge problem, the number of bridge is

(a) 5 (b) 7

(c) 9 (d) 11

6. In $C_{2,5}$, the number of edges is

(a) 7 (b) 10

(c) 3 (d) 5

7. For a graph G with 12 vertices, it $\alpha = 3$ then β is

(a) 15 (b) 36

(c) 9 (d) 3

8. The value of $r(3,3)$ is

(a) 6 (b) 9

(c) 0 (d) 3

9. If G is 5-critical then

(a) $\delta = 5$ (b) $\delta \geq 4$

(c) $\delta \geq 5$ (d) $\delta \leq 4$

10. There are only ———— types of graph G for which $\chi = \Delta + 1$?

(a) 2 (b) 3

(c) 4 (d) 5

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Explain the following with suitable examples

(i) Simple graph

(ii) Spanning subgraph

(iii) Walk

(iv) Path

(v) Cycle.

Or

(b) Prove that $\sum_{v \in V} d(v) = 2\varepsilon$ and hence show the number of vertices of odd degree is even in any graph.

12. (a) If G is a tree, prove that $\varepsilon = \gamma - 1$.

Or

(b) Prove that a vertex v of a tree G is a cut vertex of G if and only if $d(v) > 1$.

13. (a) Prove that $C(G)$ is well defined.

Or

(b) Define a maximum matching and a minimum covering. Let M be a matching k and be a covering such that $|M| = |K|$, then prove that M is a maximum making and K is a minimum covering.

14. (a) If $\delta > 0$, prove that $\alpha' + \beta' = \gamma$.

Or

(b) Prove that $r(k, l) \leq \binom{k + l - 2}{k - 1}$.

15. (a) If G is k-critical, prove that $\delta \geq k - f$.

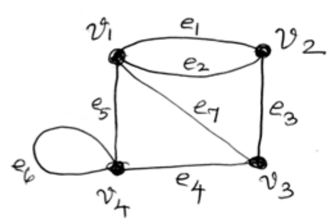
Or

(b) For any graph G, prove that $\prod_k (G)$ is a polynomial is k of degree r, with integer coefficients, leading term k^r and constant term zero and the coefficients of $\prod_k (G)$ alternative in sign.

PART C — $(5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b).

16. (a) Define the incidence and adjacency matrices of a graph. Find the two matrices for the following graph.



Or

(b) Obtain a necessary and sufficient condition for graph to be bipartite.

17. (a) Define a cut edge with an example. Prove that an edge e of G is a cut edge of G if and only if e is contained in no cycle of G.

Or

(b) State and prove Whitney's theorem for 2-connected graphs.

18. (a) Let G be a simple graph with degree sequence $(d_1, d_2, \ldots dr)$ when $d_1 \leq d_2 \leq \ldots \leq d_r$ and $r \geq 3$. Suppose that there is no value of m less than $r/2$ for which $d_m \leq m$ and $d_{r-m} < r - m$. Prove that G is Hamiltonian.

Or

(b) Prove that a matching M in G is a maximum matching if and only if G contains no M-augmenting path.

19. (a) If G is simple, prove either $\chi' = \Delta \theta \chi' = \Delta + 1$.

Or

(b) Prove that $r(k,k) \geq 2^{k/2}$.

20. (a) For any positiver integer k, prove that there exists a k-chromatic graph containing no triangle.

Or

(b) State and prove Brook's theorem.

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(7 pages) **Reg. No. :** ....

Code No. : 6379 **Sub. Code : ZMAM 33**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

Third Semester

Mathematics — Core

MEASURE AND INTEGRATION

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Let A be the set of irrational numbers in [0, 1]. Then $m^*(A)$ is

   (a) ∞ (b) 0

   (c) 1 (d) 1/2

2. If A is a measurable set of finite outer measure that is contained in B then $m^*(B \sim A) = m^*(B) - m^*(A)$. This property is known as

(a) finite subadditivity (b) translation invariant

(c) countably additive (d) excision property

3. For a function of defined on E, $f^+(x)$ is defined by

(a) $\max\{f(x) - f(x)\}$ (b) $\max\{f(x), 0\}$

(c) $\max\{-f(x), 0\}$ (d) $\min\{f(x), 0\}$

4. If for each $\varepsilon > 0$, then is an index N for which $|f - f_n| < \varepsilon$ on A for all $n \geq N$, we say that

(a) $\{f_n\}$ converges to f uniformly on A

(b) $\{f_n\}$ converges to f pointwise a.e. on A

(c) $\{f_n\}$ converges to f pointwise on A

(d) $\{f_n\} \to f$ as $n \to \infty$

5. Let f be a nonnegative measurable function on E. then for any $\lambda > 0$, $m\{x \in E / f(x) \geq \lambda\}$ is

(a) $= \lambda \int_E f$ (b) $\leq \lambda \int_E f$

(c) $\geq \lambda \int_E f$ (d) $= \frac{1}{\lambda} \int_E f$

6. Define $g_n = \chi_{(n,n+1)}$. Then $\lim_{n\to\infty} \int_{\mathbb{R}} g_n$ is

(a) 0 (b) 1

(c) ∞ (d) n

7. $[-5f]^+$ is

(a) $-5f^+$ (b) $5f^-$

(c) $5f^+$ (d) $-5f^-$

8. For all $a \le u \le v \le b$, $\int_u^v Diff_h f$ is

(a) 0 (b) $Av_h f(v) + Av_h f(u)$

(c) $Av_h f(v) - Av_h f(u)$ (d) $Av_h f(u) - Av_h f(v)$

9. A function f on $[a, b]$ is absolutely continuous on $[a,b]$ if and only if

(a) it is an indefinite integral over *[a,a]*

(b) $\int_a^b f' = f(a) - f(a)$

(c) f is differentiable almost everywhere on *(a, a)*

(d) f is differentiable

10. If $\{A, B\}$ is a Hahn decomposition for r, then r^+ and r^- are defined by

(a) $r^+(E) = r(E \cap A), r^-(E) = -r(E \cap B)$

(b) $r^+(E) = -r(E \cap A), r^-(E) = r(E \cap B)$

(c) $r^+(E) = r(E \cup A), r^-(E) = -r(E \cup B)$

(d) $r^+(E) = -r(E \cup A), r^-(E) = r(E \cup B)$

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Define a measurable set and prove that any set of outer measure zero is measurable.

Or

(b) If m is the Lebesgue measure and if $\{A_K\}_{k=1}^{\infty}$ is an ascending collection of measurable sets, prove that $m\left(\bigcup_{k=1}^{\infty} A_k\right) = \lim_{k \to \infty} m(A_k)$.

12. (a) Let f be an extended real - valued function on E. If f is measurable on E and $f = g$ a.e. on E, prove that g is measurable on E.

Or

(b) Let $\{f_n\}$ be a sequence of measurable function on E that converges pointwise a.e. on E to the function f. Prove that f is measurable.

13. (a) Let f be a bounded measurable function on a set of finite measure E. Prove that f is integrable over E.

Or

(b) State and prove Chebychev's inequality.

14. (a) Let f be integrable over E and $\{E_n\}_{n=1}^{\infty}$ a disjoint countable collection of measurable subsets of E whose union is E. Prove that $\int_E f = \sum_{n=1}^{\infty} \int_{E_n} f$.

Or

(b) Define the total variation $TV(f)$ of f on $[a,b]$. If f is a Lipschitz function on $[a,b]$, prove that f is of bounded variation of $[a,b]$.

15. (a) Define absolutely continuous functions. Prove that absolutely continuous functions are continuous. Is the converse true? Justify.

Or

(b) State and prove the Jordan decomposition theorem.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b)

16. (a) Prove that the outer measure of an interval is its length.

Or

(b) (i) State and prove that Borel - Cantelli lemma.

(ii) State the continuity properties of Lebesque measure.

17. (a) Let f and g be measurable functions on E that are finite a.e. on E. For any α and β, prove that $\alpha f + \beta g$ is measurable on E and that fg is measurable on E.

Or

(b) State and prove Egoroff's theorem.

18. (a) State and prove the bounded convergence theorem.

Or

(b) Let f and g be nonnegative measurable functions on E. For any $\alpha > 0$ and $\beta > 0$, prove that $\int_E (\alpha f + \beta g) = \alpha \int_E f + \beta \int_E g$.

19. (a) State and prove the Lebesgue dominated convergence theorem.

Or

(b) State and prove the Vitali covering lemma.

20. (a) Let the function f be absolutely continuous on $[a,b]$. Prove that f is the difference of increasing absolutely continuous functions and in particular, is of bounded variation.

Or

(b) Let γ be a signed measure on the measurable space (X,M) and E a measurable set for which $0 < \gamma(E) < \infty$. Prove that there is a measurable subset A of E that is positive and of positive measure.

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(8 pages)

Reg. No. : ....

Code No. : 6380 **Sub. Code : ZMAM 34**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

Third Semester

Mathematics — Core

TOPOLOGY — I

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. Let $X = \{a, b, c\}$, $\tau_1 = \{\phi, X, \{a\}, \{a, ab\}\}$, $\tau_2 = \{\phi, X, \{a\}, \{b, cb\}\}$. The largest topology contained in τ_1 and τ_2 is

   (a) $\{\phi, X\}$

   (b) $\{\phi, X, \{ab\}\}$

   (c) $\{\phi, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$

   (d) the discrete topology on X

2. Let $Y = (0, 1]$ be a subspace of R. Let $A = \left(0, \frac{1}{2}\right)$ the closure of A in Y is

(a) $(0, 1/2]$ (b) $[0, 1/2]$

(c) $(0, 1/2)$ (d) $(0, 1]$

3. Let $f : \{a,b,c\} \to \{1,2,3\}$ be defined by $f(a) = 2, f(b) = 3$, $f(c) = 3$. Let $V = \{1,3\}$, then $f^{-1}(V)$ is

(a) $\{a,b\}$ (b) $\{b,c\}$

(c) $\{a,c\}$ (d) $\{a,b,c\}$

4. Consider the identity functions $f : R \to R_l$ and $g : R_l \to R$, then

(a) both f and g are continuous

(b) f is continuous but g is not continuous

(c) f is not continuous but g is continuous

(d) neither f nor g is continuous

5. In R^n, we have $d(x,y) \le \sqrt{n}\rho(x,y)$, this inequality shows that

(a) $B_\rho(x,\varepsilon/\sqrt{n}) \subset B_d(x,\varepsilon)$

(b) $B_\rho(x,\varepsilon) \subset B_d\left(x,\frac{\varepsilon}{\sqrt{n}}\right)$

(c) $B_\rho(x,\varepsilon) \subset B_d(x,\varepsilon)$

(d) $B_d(x,\varepsilon) \subset B_\rho\left(x,\frac{\varepsilon}{\sqrt{n}}\right)$

6. The metric that induces the product topology on R^ω is

(a) $D(x,y) = \sup\{\bar{d}(x_i,y_i)\}$

(b) $D(x,y) = \sup\{|x_n - y_n|\}$

(c) $D(x,y) = \sup\left\{\frac{\bar{d}(x_i,y_i)}{i}\right\}$

(d) $D(x,y) = \sup\left\{\frac{i}{\bar{d}(x_i,y_i)}\right\}$

7. If X is connected then

(a) ϕ is the only subset which is both open and closed

(b) X is the only subset which is both open and closed

(c) ϕ and X are the only subsets which are both open and closed

(d) we can find a subset $A(\neq \phi, X)$ which is both open and closed

8. Which one of the following is compact in R

(a) [0, 1] (b) [0, 1)

(c) (0, 1) (d) (0, 1]

9. Which one of the following is NOT a subsequence of (x_n)

(a) $\{x_5, x_6, x_7, ...\}$

(b) $\{x_3, x_6, x_9, ...\}$

(c) $\{x_2, x_1, x_3, x_2, x_4, x_3 ....\}$

(d) $\{x_{100}, x_{200}, x_{300}, ...\}$

10. Which one of the following is not locally compact

(a) R

(b) Q

(c) R^n

(d) Every simply ordered set having the *l.u.b.* property

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Define a basis for a topology on X. If B is a basis for a topology τ on X, prove that τ equals the collection of all unions of elements AB.

Or

(b) Let Y be a subspace of X. Let A be a subset of Y. Let $\overline{A}$ denote the closure of A in X. Prove that the closure of A in Y equals $\overline{A} \cap Y$.

12. (a) If $\mathcal{B}$ is a basis for the topology on X and $\mathcal{C}$ is a basis for the topology on Y, prove that $\mathcal{D} = \{B \times C / B \in \mathcal{B}, c \in \mathcal{C}\}$ is a basis for the topology of $X \times Y$.

Or

(b) State and prove the pasting lemma.

13. (a) Let X be a metric space with metric d. Define $\overline{d} : X \times x \to R$ by $\overline{d}(x, y) = \min\{d(x, y), 1\}$. Prove that $\overline{d}$ is a metric that induces the same topology as d.

Or

(b) State and prove the sequence lemma.

14. (a) Prove that the union of a collection of connected subspace of X that have a point in common is connected.

Or

(b) Prove that every closed subspace of a compact space is compact.

15. (a) Prove that compactness implies limit point compactness.

Or

(b) Let X be a Hausdorff space. Prove that X is locally compact if and only if given x in X, and given a neighborhood U of x, there is a neighborhood V of x such that $\overline{V}$ is compact and $\overline{V} \subseteq U$.

PART C — (5 × 8 = 40 marks)

Answer ALL questions, choosing either (a) or (b)

16. (a) Define the standard topology, the lower limit topology and the K - topology on R. Find the relation between these topologies.

Or

(b) Let A be a subset of the topological space X. Let A' be the set of all limit points of A. Prove that $\overline{A} = A \cup A'$ and hence show that A is closed if and only if it contains all its limit points.

17. (a) Let X and Y be topological space; let $f: X \to Y$. Prove that the following are equivalent

(i) f is continuous

(ii) for every subset A of X, one has $f(\overline{A}) \subseteq \overline{f(A)}$

(iii) for every closed set B of Y, the set $f^{-1}(B)$ is closed in X.

Or

(b) Let $f: A \to \underset{\alpha \in J}{\pi X_\alpha}$ be given by the equation $f(a) = (f_\alpha(a))_{\alpha \in J}$, where $f_\alpha : A \to X_\alpha$ for each α. Let πX_α have the product topology. Prove that the function f is continuous if and only if each function f_α is continuous.

18. (a) Prove that the topologies on R^n induced by the Euclidean metric d and the square metric ρ are the same as the product topology on R^n.

Or

(b) State and prove the uniform limit theorem.

19. (a) Prove that a finite Cartesian product of connected spaces is connected.

Or

(b) State and prove the tube lemma.

20. (a) Let X be a metrizable space. If X is sequentially compact, prove that X is compact.

Or

(b) Let X be a space. If X is locally compact Hausdorff prove that there exists a space Y satisfying

(i) X is a subspace of Y

(ii) $Y - X$ consists of a single point

(iii) Y is a compact Hausdorff space.

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(7 pages)

Reg. No. : ....

Code No. : 6381 **Sub. Code : ZMAE 31**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

Third Semester

Mathematics

Elective — ALGEBRAIC NUMBER THEORY

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 1 = 10$ marks)

Answer ALL questions.

Choose the correct answer :

1. Which of the following Diophantine equation cannot be solved?

   (a) $6x + 51y = 22$ (b) $33x + 14y = 115$

   (c) $14x + 35y = 93$ (d) $11x + 13y = 21$

2. The linear Diophantine equation $ax + by = c$ has a solution if and only if ————

   (a) $\gcd(a,c) | b$ (b) $\gcd(a,b) | c$

   (c) $\gcd(c,b) | a$ (d) $c | \gcd(a,b)$

3. A linear combination of integers a and b is

(a) ab

(b) $\frac{a}{x}+\frac{b}{y}$, x and y are integers

(c) $ab=1$

(d) $ax+by$, x and y are integers

4. Let a and b be integers, not both zero. Then a and b are relatively prime iff there exists integers x and y such that

(a) $1=ax+by$

(b) $2=ax+by$

(c) $ab=ax+by$

(d) $a-b=ax+by$

5. The number of prime is

(a) finite

(b) infinite

(c) uncountable

(d) 1729

6. Two integers a and b, not both of which are zero, are said to be relatively prime if

(a) $\gcd(a,b)=a$

(b) $a\mid b$

(c) $\gcd(a,b)=1$

(d) $b\mid a$

7. If a is a solution of $P(x) \equiv 0 (\bmod n)$ and $a \equiv b (\bmod n)$, then

(a) ab is also a solution

(b) $a+b$ is also a solution

(c) $a-b$ is also a solution

(d) b is also a solution

8. In n is an odd pseudo prime, then $2^n - 1$ is

(a) pseudo prime (b) prime

(c) irrational (d) not pseudo prime

9. If m and n are relatively prime integers then $\varphi(mn) =$ ——————

(a) $\varphi(m) + \varphi(n)$ (b) $\varphi(m) / \varphi(n)$

(c) $\varphi(m) - \varphi(n)$ (d) $\varphi(m)\varphi(n)$

10. If p is a prime and a is any integer then $a^p - a$ is

(a) a multiple of p^2 (b) a multiple of $p-1$

(c) a multiple of $2p$ (d) a multiple of p

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).
Each answer should not exceed 250 words.

11. (a) Find all solutions in positive integer $5x + 3y = 52$.

Or

(b) If u and v are relatively prime positive integers whose product uv is a perfect square, then prove that u and v are both perfect squares.

12. (a) Prove that the equation $x \; + 2y^3 + 4z^3 = 9w^3$ has no nontrivial solution.

Or

(b) Determine the solution of the Diophantine equation $x^2 + 3y^2 + 5z^2 + 7xy + 9yz + 11zx = 0$.

13. (a) For any positive real number x, prove that $\langle a_0, a_1, ..., a_{n-1}, x \rangle = \dfrac{xh_{n-1} + h_{n-2}}{xk_{n-1} + k_{n-2}}$.

Or

(b) Let $\theta = \langle a_0, a_1, a_2, ... \rangle$ be a simple continued fraction. Then $a_0 = [\theta]$. Further more if θ_1 denotes $\langle a_0, a_1, a_2, ... \rangle$ then prove that $\theta = a_0 + 1/\theta_1$.

14. (a) Let ξ denote any irrational number. If there is a rational number $\frac{a}{b}$ with $b \geq 1$ such that $\left|\xi - \frac{a}{b}\right| < \frac{1}{2b^2}$ then prove that $\frac{a}{b}$ equals one of the convergents of the simple continued fraction expansion of ξ.

Or

(b) If an irreducible polynomial $p(x)$ divided a product $f(x)g(x)$, then prove that $p(x)$ divides at least one of the polynomials $f(x)$ and $g(x)$.

15. (a) Let m be a negative square-free rational integer Prove : the field $\mathbb{Q}(\sqrt{m})$ has units ± 1, and these are the only units except in the cases $m = -1$ and $m = -3$. The units for $\mathbb{Q}(i)$ are ± 1 and $\pm i$. The units for $\mathbb{Q}\sqrt{-3}$ are ± 1, $(1 \pm \sqrt{-3})/2$ and $\left(-1 \pm \sqrt{-3}/2\right)$

Or

(b) Prove that the integers of any algebraic number field form a ring.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b)
Each answer should not exceed 600 words.

16. (a) Find all solutions in integers of the simultaneous equations $20x + 44y + 50z = 10$, $17x + 13y + 11z = 19$.

Or

(b) Find all integers x and y such that $147x + 258y = 369$.

17. (a) Determine whether the equation $3x^2 + 5y^2 + 7z^2 + 9xy + 11yz + 13zx = 0$.

Or

(b) Prove that the equation $y^2 = x^3 + 7$ has no solution in integers.

18. (a) Prove that the equations $h_i k_{i-1} - h_{i-1} k_i = (-1)^{i-1}$ and $r_i - r_{i-1} = \frac{(-1)^{i-1}}{k_i k_{i-1}}$ hold for $i \geq 1$.

Or

(b) Prove that the value of any infinite simple continued fraction $\langle a_0, a_1, a_2, ... \rangle$ is irrational.

19. (a) If a/b is a rational number with positive denominator such that $\left|\xi - \frac{a}{b}\right| < \left|\xi - \frac{h_n}{k_n}\right|$ for some $n \geq 1$, then $b > k_n$. In fact if $|\xi b - a| < |\xi k_n - h_n|$ for some $n \geq 0$, then prove that $b \geq k_{n+1}$.

Or

(b) Prove that any periodic simple continued fraction is a quadratic irrational number and conversely.

20. (a) The norm of a product equals the product equals the product of the norms, $N(\alpha\beta) = N(\alpha)N(\beta)$. $N(\alpha) = 0$ iff $\alpha = 0$. The norm of an integer in $\mathbb{Q}\left(\sqrt{m}\right)$ is a rational integer. If γ is an integer in $\mathbb{Q}\left(\sqrt{m}\right)$, then prove that $N(\gamma) = \pm 1$ iff γ is a unit.

Or

(b) Prove that the fields $\mathbb{Q}\left(\sqrt{m}\right)$ for $m = -1, -2, -3, -7, 2, 3$, are Euclidean and so have the unique factorization property.

———————

(6 pages)

Reg. No. : ....

Code No. : 6382 **Sub. Code : ZMAE 32**

M.Sc. (CBCS) DEGREE EXAMINATION,
NOVEMBER 2022.

Third Semester

Mathematics

Elective — CALCULUS OF VARIATIONS AND INTEGRAL EQUATIONS

(For those who joined in July 2021 onwards)

Time : Three hours Maximum : 75 marks

PART A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer :

1. A necessary condition for the existence of a maximum or minimum at a point x_0 inside (a,b) is that ——————

(a) $\frac{dy}{dx} < 0$ at x_0 (b) $\frac{dy}{dx} > 0$ at x_0

(c) $\frac{dy}{dx} = 0$ at x_0 (d) $\frac{dy}{dx} \neq 0$ at x_0

2. An extremal which satisfies the appropriate end conditions is often called a ————

(a) Stationary function (b) Extremal function

(c) Maximum function (d) Minimum function

3. Derivative of the variation with respect to an independent variable is ———— the variation of derivative.

(a) Opposite (b) Same as

(c) Reciprocal (d) Twice

4. The stationary function for an integral function is one for which the variation of integral is ————

(a) 0 (b) 1

(c) –1 (d) ∞

5. Fredholm equation is ————

(a) $\alpha(x)y(x) = F(x) + \lambda \int_a^b K(x, \xi) y(\xi) d\xi$

(b) $\alpha(x)y(x) = F(x) - \lambda \int_a^b K(x, \xi) y(\xi) d\xi$

(c) $\alpha(x)y(x) = \lambda \int_a^b K(x, \xi) y(\xi) d\xi$

(d) $\alpha(x)y(x) = -\lambda \int_a^b K(x, \xi) y(\xi) d\xi$

6. Volterra equation become integral equation of first kind if ——————

(a) $\alpha \equiv 0$ (b) $\alpha \equiv 1$

(c) $\alpha \equiv -1$ (d) $\alpha \equiv y$

7. If a function $F(x)$ is identically zero in $y(x) = F(x) + \lambda \int_a^b K(x, \xi) y(\xi) d\xi$, the integral equation is said to be ——————

(a) Non homogeneous (b) Homogeneous

(c) linear (d) Constant

8. The determinant $\Delta = |I - \lambda A|$ vanishes then the integral equation $y(x) = F(x) + \lambda \int_a^b K(x, \xi) y(\xi) d\xi$ has —————— solution.

(a) only one solution

(b) infinitely many solution

(c) no solution

(d) finite

9. The characteristic numbers of a Fredholm equation with a real symmetric kernel are all ——————

(a) real (b) zero

(c) imaginary (d) integers

10. A Fredholm equation with a non symmetric kernel process characteristic numbers which are not ————

(a) real (b) zero

(c) imaginary (d) integers

PART B — $(5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Explain about Lagrange multipliers.

Or

(b) Prove that the shortest distance between two points in a plane is a straight line.

12. (a) Explain about the variational notation.

Or

(b) Obtain the partial differential equation satisfied by the equation of a minimal surface, the surface passing through a simple closed curve $\mathcal{C}$ and have the minimum surface area bounded by $\mathcal{C}$.

13. (a) Write the definition and properties of Green's function.

Or

(b) Transform the problem $y''+xy=1$, $y(0)=0,\ y(l)=1$ to an integral equation.

14. (a) Discuss about the influence function.

Or

(b) Find the conditions which determined influence function for the small deflections of a string fixed at the points $x = 0$ and $x = l$ under a loading distribution of intensity.

15. (a) Show that the characteristic numbers of a Fredholm equation with a real symmetric kernel are all real.

Or

(b) Determine the continuous solution of non homogeneous Fredholm equation of second kind.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions, choosing either (a) or (b)

16. (a) Derive Euler's equation.

Or

(b) Discuss natural boundary condition and transition condition.

17. (a) Derive specializations of Green's theorem.

Or

(b) Explain variable end points.

18. (a) Derive relations between differential and integral equations.

Or

(b) Derive Fredholm equation of second kind.

19. (a) Solve Fredholm equations with separable kernels.

Or

(b) Discuss the consistency of integral equation

$$y(x) = \lambda\int_0^1 (1 - 3x\xi)y(\xi)d\xi + F(x).$$

20. (a) Discuss iterative methods for solving equations of the second kind.

Or

(b) Derive Hilbert - Schmidt theory.

———————